OPTIMAL REPLACEMENT POLICIES FOR SATELLITE CONSTELLATIONS

THESIS

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Abstract

This work considers the problem of finding optimal replacement policies that minimize the expected total cost of maintaining a satellite constellation. The problem is modeled using discrete-time Markov decision processes to determine the replacement policy by allowing the satellite constellation to be in one of a finite number of states at each decision epoch. The constellation stochastically transitions at each time step from one state to another as determined by a set of transition probabilities. At each decision epoch, a decision maker chooses an action from a set of allowable actions for the current system state. A cost associated with each possible action is determined by the number of satellites purchased, launched, or held in storage, as well as the operational capability of the constellation. The system is evaluated for a given time horizon using the standard Policy Evaluation Algorithm of Markov decision processes (stochastic dynamic programming) to determine the optimal replacement policy and the minimum expected total cost. Example problems using notional data are presented to demonstrate the solution procedures. Sensitivity analysis of problem parameters is performed to investigate their impact on the minimum expected total cost of operating the constellation over a specified time horizon.

OPTIMAL REPLACEMENT POLICIES
FOR SATELLITE CONSTELLATIONS

1. Introduction

1.1 Background

The United States Air Force maintains a wide variety of satellite constellations used for such purposes as navigation, communications, weather, early warning, and intelligence collection. As the satellites that compose these constellations deteriorate and tend towards failure, their replacement is essential. Satellites that have been in service for any length of time suffer some amount of degradation, but most are still useful and able to accomplish their mission to varying degrees, until the degradation is substantial enough that the satellite is deemed unable to satisfactorily accomplish the mission. Ideally, each satellite would be replaced just prior to its failure to prevent degradation of the constellation's ability to perform its mission, while also maximizing the useful life of each satellite. Degrading the capability of a *constellation* to perform its mission can have grave consequences to national security, especially considering the mission of the constellation, the state of world affairs, and military actions in progress.

One way to ensure a replacement is available in the event of satellite failure is to maintain spare satellites on-orbit. However, satellites are extremely expensive to build and launch making this approach unrealistic except for the most critical assets. The financial cost of maintaining a satellite constellation must be weighed against the "cost" in terms of national security to have a loss in mission capability due to an on-orbit failure of a satellite when no replacement is available. It is therefore desirable to prescribe a satellite replacement policy which balances the

need to maintain constellation mission capabilities and avoid undue replacement costs by minimizing the monetary and security costs to the nation over the lifetime of the satellite system.

In a national defense context, finding such an optimal replacement policy is important to the United States government and its citizens because the policy minimizes the cost of providing for the security of the nation. Optimal satellite replacement policies are also important to companies in the private sector, such as telecommunications or satellite television providers. While a company may not be directly concerned with national security, customer satisfaction and good will are important performance metrics for the survival of any company. Competitive firms desire to minimize the overall costs while maintaining a high level of customer satisfaction.

The money saved from implementing such policies can be used to make improvements to the satellite constellation's capabilities or robustness, or the money could be directed to different projects altogether, making some previously unfunded or underfunded programs possible. Monetary savings on government systems could also be passed along to taxpayers in the form of tax breaks. In the case of a private firm, savings can be passed to investors in the form of dividends or to customers in the form of lower prices, creating a competitive advantage for the firm.

Research in the areas of optimal replacement policies and the modeling of satellite constellations are relevant to the problems presented here. Optimal replacement problems have been studied extensively in the stochastic operations research literature. Solutions for general mechanical or electrical systems are often found using renewal theory. Open source literature on the modeling of satellite constellations, however, is much more sparse. Only a few journal articles have been published addressing different approaches to modeling satellite constellations.

Problems involving the modeling of satellite constellations generally use Monte-Carlo simulation to determine actions for maintaining constellations, predicting

satellite reliability, or assessing satellite constellation availability [15] [21] [23]. The models themselves are sometimes set up purely as simulation models based on satellite failure times or the satellite constellations are modeled using a network of queues. In both cases, Monte-Carlo simulation is generally used to analyze the model due to issues of complexity. When using Monte-Carlo simulation, one run of the simulation results in one data point of information, or one possible outcome, of all the possible outcomes that could occur in the experiment. To appropriately interpret the results, the experiment must be replicated numerous times and the results subjected to careful statistical analysis to make the correct inferences regarding system performance. Monte-Carlo simulation can be used to compare two or more models, but does not allow the user to determine if an evaluated model is optimal. For this reason, an analytical solution is preferred because such a solution can be shown to be optimal.

This thesis approaches the problem of finding an optimal satellite replacement policy from an analytical point of view. The specific approach taken to solve the problem is to analytically model satellite constellations and then use Markov decision processes to determine the optimal replacement policy of the satellites. In this context, optimal means the minimum expected total cost over the time-horizon evaluated. Using Markov decision processes to model the stochastic evolution of a deteriorating satellite constellation is useful because, for the given inputs, they provide a provably optimal replacement policy under certain assumptions. Such an optimal policy avoids a potential source of error because there is no need to make inferences from the model outputs regarding the optimality of the solution.

1.2 Problem Definition and Methodology

In this thesis, a satellite constellation is stochastically modeled and analyzed to find a satellite replacement policy which minimizes the expected monetary and opportunity costs (e.g. national security costs and costs associated with gain or loss of customer satisfaction) of maintaining the constellation. While certain budgetary

constraints may be imposed on the implementation of a policy (e.g. a maximum annual budget) the optimal policy found herein does not take into account budgetary constraints and is useful for establishing budgets or lobbying for funding levels that minimize costs over the lifetime of the system.

The research objectives of this thesis are to analytically model satellite constellations, to find optimal satellite replacement policies for maintaining constellations, and to study how changes to model parameters affect the minimum expected total cost of maintaining a constellation. The satellite constellations are analytically modeled using stochastic processes, specifically discrete-time Markov chains in the context of Markov decision processes. Optimal replacement policies are found by using the policy evaluation algorithm of Markov decision processes. Sensitivity analysis is performed to investigate the impact of model parameters on the minimum expected total cost of maintaining a constellation.

The proposed models are created using finite-horizon Markov decision processes. The optimal replacement policy for minimizing the many costs associated with maintaining a satellite constellation is found by using a policy evaluation algorithm. The policy obtained using this technique is optimal for minimizing the total expected cost of maintaining the constellation, subject to the imposed assumptions. The value of the minimum expected total cost is also provided by the policy evaluation algorithm and is the same value that can be derived by solving the optimality equations along with the boundary condition. The fact that the value from the policy evaluation algorithm agrees with the results from the optimality equations shows that the policy evaluation algorithm does indeed result in the optimal value of the replacement problem.

The main contribution of this research is to provide a sound foundation upon which a more detailed analytical analysis can be based in the future. Establishing an analytical model for satellite replacement is a significant contribution because it allows optimal replacement policies to be found under some mild problem assump-

tions. Moreover, the analytical approach to the problem circumvents the need for costly and time-consuming simulation studies. The resulting policies are also useful in determining budget inputs because the expected cost per time period can be determined. This thesis also provides a means in which sensitivity analysis may be easily performed.

1.3 Thesis Outline

Chapter 2 provides a review of the literature addressing similar problems. Chapter 3 provides an overview of Markov decision processes and presents conditions for the existence of an optimal replacement policy over a finite time horizon. Chapter 4 presents notional numerical examples demonstrating the applicability of these models. Chapter 5 reviews the contributions and limitations of this work and discusses recommendations and future research directions.

2. Literature Review

There are two main areas of literature that are relevant to this problem. The first area is that of optimal replacement for degrading systems. Optimal replacement problems have been studied a great deal in the stochastic operations research literature. The second area is the mathematical modeling of satellite constellations. Three specific models will be reviewed in detail.

2.1 Optimal Replacement Models

Survey papers by McCall [27], Pierskalla and Voelker [34], and Valdez-Flores and Feldman [38] review the optimal replacement literature from the early 1950s through the late 1980s. These papers provide a good summary of optimal replacement literature and models through that time.

A seminal work by Barlow and Proschan [6] introduces many single-unit models. Much of the later literature is based on this work. Barlow and Proschan [6] describe an array of optimal maintenance policies, including age replacement models which assume that spares are always available. Nakagawa and Osaki [32] extend this model to allow for the case where a spare is not always available. They model the lead time required to obtain a replacement as a random variable. In their model, Nakagawa and Osaki order the new spare immediately after each replacement. By ordering the spare immediately, the spare may arrive well before the replacement takes place. Storing the spare from the time it arrives until the replacement takes place results in a holding cost which can be quite expensive. It, therefore, may be better to delay ordering the replacement. Mine and Kawai [28] address the case of delaying the order of the replacement to minimize the holding cost.

Barlow and Proschan ([6], page 18), also present another topic that is important to the development of the model herein. They define *interval reliability* as the probability that the system will continue to operate from some time to some specified

time in the future. The difference in times is the interval. Interval reliability is an important concept to the satellite replacement models presented in Chapter 3.

Mine and Nakagawa [29] also do work on interval reliability, specifically when the distribution is exponential. They use a renewal theory approach to find a preventative maintenance policy that maximizes the interval reliability of the system under evaluation. In this work, maintenance or a repair can be considered analogous to a replacement in the satellite problem. This is allowed because the authors use a repair to correct a system failure in the same way that a replacement would correct a failure.

Aven and Bergman [3], Dekker [13], and Aven and Dekker [4] present work on a general structure for optimal replacement problems. These general frameworks are based on the application of renewal theory. The goal in this approach is to determine the optimal replacement time with which to optimize the expected total cost of the system.

Aven and Bergman [3] formally describe both a continuous-time and a discrete-time structure. They claim that their structure can be applied to a large class of replacement models. The authors present a general approach to minimizing the expected total discounted cost as well as the long-run expected average cost per unit time. This general approach involves conditions and assumptions that are independent of specific problems. Both the continuous-time and the discrete-time frameworks are developed by thorough definitions of the probability space and characteristics of the applicable measure processes, such as the failure and repair/replacement processes. A derivation of the optimal stopping time is provided.

Dekker [13] deals largely with maintenance activities and allows penalty cost functions to be derived for deviating from the optimal maintenance interval. The author claims that the penalty costs can be used to set priorities for action selection. He provides penalty functions for short-term, long-term, and permanent shifts from

the optimal policy. It is also claimed that the penalty costs can assist with production planning.

Aven and Dekker [4] extend the types of models addressed by Dekker [13]. This paper is also based on renewal theory. The authors state that the framework presented in this paper is a simpler version of the framework presented in [3]. After presenting their general framework and assumptions, the authors offer examples of how to apply the framework to several types of problems. Some of the problems addressed are opportunity-based age replacement problems, opportunity-based block replacement problems, and minimal repair models.

The literature discussed thus far primarily deals with single-unit systems. These problems were largely addressed by the use of renewal theory. The body of literature on multi-unit models, although not as developed as literature on single-unit models, has been growing since the mid-1980's. Analytical modeling of multi-unit systems typically relies on the application of dynamic programming. Literature on optimal replacement policies of multi-unit non-repairable systems is limited, possibly due to the dimensionality of the state space for such problems. The following articles address multi-unit optimal replacement models.

Ben-Ari and Gal [7] and Gal [20] present a multi-unit model for which an optimal replacement policy is found. The model is complicated by the fact that there is an interaction between the items in the system. Gal [20] gives, as an example of this type of interaction, the case in which a lead time for a replacement order is incorporated. Ben-Ari and Gal [7] use a dynamic programming approach to find an optimal replacement policy. To circumvent the state space explosion of such problems, the method combines computer simulation and dynamic programming. This method is called the Parameter Iteration Method to differentiate it from the Value Iteration Method commonly used in dynamic programming.

While Ben-Ari and Gal [7] present an application of the Parameter Iteration Method, the focus of Gal [20] is the method itself. The Parameter Iteration Method

method was first introduced by Gal [19]. The method is applied when, at each time period t, the optimal value (see Section 3.1 for a review of Markov decision processes/stochastic dynamic programming) is approximated by a function, from a user-determined set of *admissible* functions, that depends on parameters of the state variables [20].

This optimal return function is evaluated, for each time period, by performing dynamic programming recursions at enough states to determine the function. Gal [20] claims that, for Markov decision problems with a small amount of uncertainty, the parameter iteration fits well with the use of simulation. Furthermore, he states that a policy considered to be "reasonable" is used to simulate the sequence of states followed by some number of realizations of the policy. The return function is then approximated by one of the admissible functions for the states visited by these realizations. The approximations are accomplished by beginning in the final time period being evaluated and working backwards toward the initial time period. Each iteration of the this process results in a new policy that is an improvement over the previous policy. New realizations are then simulated using the new policy and the process is repeated until the return function converges.

Gal [19] points out the Parameter Iteration Method is not automatic and requires that the user have a good understanding of the system being evaluated in order to determine the class of admissible functions for the return function. Ben-Ari and Gal [7] refer to this approximation to the optimal return policy as a practical solution to the problem.

Flynn, *et al.* [17] present an optimal replacement model for a multi-component reliability system. The goal of their model is to find the optimal balance between the cost of component replacement and the cost of system failure. At the beginning of each time period, a decision is made whether to replace any failed components. Replacement components are assumed to always be available. The problem is formulated as a stochastic dynamic program (Markov decision process). To address

the problem of state space explosion, the authors restrict their attention to critical component policies which allow the replacement of a system component only if the component has failed and is considered to be critical for the operation of the system. Their model assumes that the components are either operational or failed and evaluate the system over an infinite time horizon. The model presented by these authors is a multi-unit model that does not mandate the replacement of failed components. In this thesis, a constellation is analogous to the system and a satellite is analogous to a component. The models in Chapter 3, however, do not assume that the system (constellation) itself fails when a component (satellite) is failed, but instead a penalty cost is charged whenever components of the systems are failed, and thus, performance is degraded.

Chung and Flynn [10] extend their earlier study to find optimal replacement policies for k-out-of-n systems. A k-out-of-n system is one which consists of n components and requires at least k of the components to be operational for the system to function. This paper uses the same assumptions as Flynn *et al* [17] expect the problem is extended to find the optimal replacement policy when k-out-of-n independent components must function for the system to be operational. This optimal replacement policy is found using a dynamic programming formulation.

Chung and Flynn [11] improve on that work by presenting a more efficient branch-and-bound algorithm that finds optimal replacement polices for k-out-of-n systems. Flynn and Chung [18] continue their work in this area by developing a branch-and-bound technique for consecutive k-out-of-n systems. For a consecutive k-out-of-n system at least k consecutive components must be operational (the operational components are not separated by any failed components) for the system to function.

2.2 Satellite Constellation Models

Jacobs *et al.* [23] explain the software program Operational Constellation Availability and Reliability Simulation (OSCARS). OSCARS is used by Air Force Space Command to analyze and compare satellite constellation replenishment strategies (also known as policies). OSCARS uses Monte-Carlo simulation to estimate when satellites need to be launched to maintain a specified number of operational satellites.

OSCARS has two main functions: Generate Launch Schedule and Evaluate Launch Schedule. The Generate Launch Schedule function generates a launch schedule based on data from several databases containing information about existing satellites, planned launches, and the inventory levels of both replacement satellites and the boosters needed to launch them. This function identifies how many satellites need to be launched and when they should be launched to maintain a constellation with the specified number of operational satellites. The Evaluate Launch Schedule function evaluates a generated launch schedule to determine how many operational satellites are available, at any specified time, when following the schedule produced by the Generate Launch Schedule function. The number of operational satellites maintained by following the generated schedule is compared to the required number of operational satellites to determine the performance of the schedule being evaluated.

The events in OSCARS are driven by satellite failures. OSCARS uses satellite failure distributions specified by the user to estimate when a satellite failure will occur. The satellite failure distribution can be represented by a single probability distribution or it can be modeled by allowing separate probability distributions to represent the phases (Infant Mortality, Useful Life, and Wearout) of the satellite's lifetime. The Infant Mortality phase is modeled by a user prescribed probability of infant mortality. According to Jacobs *et al.* [23] the infant mortality "represents the

percent of time that the satellite will fail between its launch date and the end of the first month of operation."

The Useful Life phase of the failure distribution is modeled by the Weibull distribution. The age of an existing satellite at the beginning of the simulation is taken into account by OSCARS. This age determines where the satellite is in relation to the failure curves. Jacobs *et al.* [23] state that "The Weibull distribution has historically been selected to model satellite failures."

The Wearout phase is modeled in four different ways. The first way is described as a fixed cutoff or cliff where the satellite failure occurs either before or at a specified date. The other ways the Wearout phase is implemented includes the use of the Rayleigh distribution, the normal distribution, and the normal distribution with a fixed cutoff date, which is a combination of the normal distribution and the fixed cutoff methods.

Outputs from OSCARS are divided by the function that produced them. The main output of the Generate Launch Schedule function is Launch Need Date. A Launch Need Date is a date such that, in a specified percentage of the simulation replications a launch was required by that date. For example, if in 10 percent of the replications, a launch was required by some date, that date would be a 10 percent Launch Need Date. The Generate Launch Schedule function also produces statistics on satellite and booster inventory demands. The purpose of the Evaluate Launch Schedule function is to determine how many operational satellites will be available during the period of time covered by the simulation. The main outputs of this function are a graph of the median number of operational satellites available during the simulation and a graph of the probability of having at least the required number of satellites at any given time during the simulation.

Hansen [21] makes reliability predictions for satellite constellations by focusing on satellite subsystems. Hansen states that reliability for electronic components is normally defined as a probability of "success", or the probability that the system

will perform its intended function for some given period of time. He then raises the point that the definition of a success needs to be clarified when talking about satellite constellation reliability. Satellites are built with many redundancies because most satellites cannot be repaired once they are launched. This makes the definition of success more complicated. Should a success be when all components are working or a certain function is being accomplished? In this thesis, reliabilities consider the functionality of a system. It is left to the reader to determine what method is most appropriate for their purposes.

Hansen [21] discusses the assumption adopted from MIL-HDBK-217 , that the lifetime of all satellite subsystem components are distributed exponentially, is also discussed. Hansen [21] claims this assumption does not accurately represent the actual reliability of the components. Instead he offers a five parameter distribution that is a linear combination of an exponential distribution used to model infant mortality of a component and a three parameter Weibull distribution that is used to model the remaining lifetime of the component. Hansen [21] claims this five parameter distribution is a realistic alternative to the exponential component lifetimes assumed above.

Hansen [21] also provides an example of the redundancy measures for a subsystem of a satellite. He claims that analytically determining the reliability of subsystem functions is extremely complicated, if not impossible, and uses Monte-Carlo simulation to carry out the analysis of subsystem reliabilities.

Ereau and Saleman [15] study the availability of satellite constellations by modeling the constellations using stochastic Petri nets. The authors claim that availability analysis during the development phase of a satellite constellation provides important information that can be used for system definition, such as determining optimal placement of the satellites and maintenance strategies. They also claim that availability analysis helps minimize global costs. The authors state that the use of

Petri nets is better able to handle the combinatoric explosion of the number of states than other types of models.

The authors state that classic methods, such as Reliability Block Diagrams and Fault Trees, are good at representing dependency links between system components, but are poor for modeling complex processes, such as resource sharing. They go on to say that Markov chains can be used to model any type of finite-state process by completely enumerating the system states. They claim that, for satellite constellations, the state space grows quickly and this method become intractable. This problem is handled in Chapter 3 by making reasonable assumptions to limit the state space of the systems being studied. The speed and memory capabilities of modern computers helps minimize this concern. Of course, it is always possible to make a model that is too large for current computer capacities. For most reasonably sized constellations (constellations with as many satellites as Iridium or the proposed Teledesic system are most likely beyond the range of reasonably sized), the state space of the problem can be held in check by these assumptions.

The model of Ereau and Saleman [15] is based on a Low Earth Orbit (LEO) constellation with p orbital planes that have k out of n satellites each. The system has both a space segment and ground logistic support segment. The ground segment has c independent production lines that each produce k satellites. The ground segment also has capacity to store s sets of one launcher and k satellites. There are l independent launching areas used to launch the satellites. The space segment allows both nominal (active) and standby satellites to be in orbit.

The model is considered to begin with no satellites in orbit and undergoes an initialization phase to get the satellites into orbit. Each launcher is assumed to launch k satellites and so it takes at least p launches to populate each orbital plane. In the event of a launch failure, a new launcher and set of k satellites must be ordered, thus delaying the completion of the initialization phase. During this phase no standby satellites are launched into orbit.

To simplify the modeling process, Ereau and Saleman [15] use the same model for initializing the system as well as maintaining the constellation. This results in k satellites being launched in every replacement launch. When the first satellite fails, k satellites are launched into that orbital plane. One of the satellites will serve as a replacement while the other $k-1$ satellites are put into standby mode. As satellites in that orbital plane continue to fail, the standby satellites are activated to replace the failed satellites. When no more standby satellites remain in that plane, the next satellite failure of the plane will result in k more satellites being launched into that plane. While satellites are on orbit in standby mode they are subject to a satellite failure rate that is lower than the failure rate of the active satellites.

The model is implemented by using a global Petri net made up of smaller Petri nets for the different model segments. For example, there is a separate network for each orbital plane. These individual networks model each state and transition that can occur to the satellites. The ground segment is also made up of its own network. Another network takes care of interfacing the ground and space segment networks.

Ereau and Saleman [15] explain, that to be used for quantitative analysis, Petri nets must be extended to incorporate the use of time. This extension of Petri nets is called Stochastic Timed Petri Nets. Analytical results with this type of network are possible, but the issue of state space explosion still exists. The authors state that if analytical methods were used, the p orbit plane models would have more than $160,000$ states for an example that has three orbital planes each with two active satellites and a slot for a standby satellite. By comparison, the model presented in Section 3.4 would have $4,608$ states for a constellation with nine satellites.

Thus, in order to obtain qualitative results for their model, the authors resort to Monte-Carlo simulation. The authors claim that their symbolic modeling of the satellite constellation with the Petri nets allows broad sensitivity analysis for the input parameters without having to change the model. Ereau and Saleman [15] conclude by providing examples of outputs from their model. The Central Limit

Theorem is applied to determine confidence intervals for mission availability over the lifetime of the mission. They also present a figure showing the probability that a given number of satellites are ordered during the evaluated time period.

The satellite replacement problem presented in this thesis is a multi-unit system with stochastically deteriorating components. Under normal conditions, satellites are not repairable and must be replaced. A large portion of the optimal replacement literature deals with determining maintenance times and finally replacement of repairable systems in order to minimize cost or maximize availability. Much of this literature cannot be directly applied to finding replacement policies for non-repairable systems. However, several models from the optimal replacement literature were reviewed that could be applied to satellites. Much of this literature concerned the use of renewal theory and was more closely aligned with the modeling of a single satellite system. Literature regarding the optimal replacement policy for multi-unit systems typically used dynamic programming to evaluate the systems.

The work by Gal [19], Ben-Ari and Gal [7], and Gal [20] addressed multi-unit systems, but found only an approximation to the optimal policy. The goal of this research is to analytically model the satellite constellations and to find a provably optimal replacement policy to minimize the expected total cost of maintaining the constellation. Flynn *et al.* [17], Chung and Flynn ([10], [11]) and Flynn and Chung [18] simplify the system by only considering the critical components. They assume that if any of these critical components fail, the system also fails. This differs from the problem addressed here in that, when an active satellite fails, the constellation is degraded, but still capable of providing some usefulness so long as at least one satellite remains operational.

The work by Jacobs *et al.* [23] and Ereau and Saleman [15] model satellite constellations. Hansen [21] addresses modeling satellite subsystems which has some similarities to the modeling of satellite constellations. All three of these articles use

Monte-Carlo simulation to analyze their models. A provably optimal replacement policy cannot be found by this method.

This thesis uses Markov decision processes (which are solved as stochastic dynamic programming problems) to provide a general analytical model to find optimal replacement policies for satellite constellations which minimize the expected total cost of maintaining the constellation. Markov decision processes are used to solve the models because the provably optimal replacement policy can be found in this way. Moreover, the resulting policy vector can be easily implemented by a decision maker. A review of Markov decision processes is provided at the beginning of Chapter 3.

3. Formal Model Description

The problem, as discussed in Chapter 1, is to find the optimal replacement policy which minimizes the expected total cost (monetary and opportunity costs) of maintaining a satellite constellation. A policy meeting these criteria balances the need to have a fully operational satellite constellation, capable of fulfilling its intended mission, with the need to limit the funding required to maintain the constellation. The optimal replacement policy can be found analytically by applying finite-horizon Markov decision processes. A review of Markov decision processes is presented next to provide a framework for the satellite replacement problem.

3.1 A Review of Markov Decision Processes

Mine and Osaki [30] define a Markov decision process as "a sequential decision process on a discrete-time Markov chain" where a discrete-time Markov chain (DTMC) is a stochastic process with specific properties. Kulkarni [26], page 16, defines a stochastic process $\{X_n,\ n \geq 0\}$ to be a DTMC with state space S, where X_n is the state of the system at time n, if for all $n \geq 0,\ X_n \in S$ and the Markov, or memoryless, property holds (i.e. the history of the process is contained in the current state of the system, so that only the current state of the system needs to be considered). A DTMC transitions from state to state at discrete time points. A sample path showing how a DTMC might transition is shown in Figure 3.1. The state to which the process transitions is determined by the transition probabilities. The state transitions are determined by the system being modeled (state transitions which are not possible for a given system have transition probabilities equal to zero). The possible states and the transition probabilities determine how the system evolves over time and are also dependent on the system being modeled.

Puterman [35] states that Markov decision processes are "also referred to as stochastic dynamic programs or stochastic control problems." White [39], page 1,

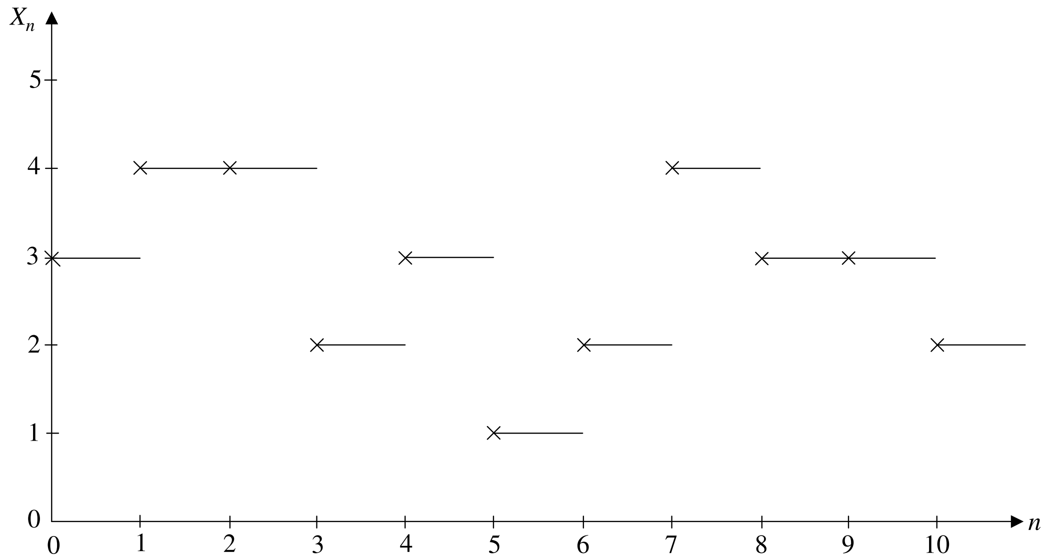

Figure 3.1 Possible DTMC sample path.

adds that the essential objective of Markov decision processes is to determine which of the possible actions is optimal for each state. This review of Markov decision processes is divided into three parts. The first part discusses the formulation of a Markov decision process model. The second part addresses optimality criteria for Markov decision processes. The final section covers the backward induction algorithm and linear programming formulations of a finite-horizon Markov decision processes. This review follows, in large part, from the excellent treatment by Puterman [35].

3.1.1 Formulation of a Markov Decision Process

Puterman [35], pages 17-22, clearly lays out the components of a Markov decision process model whose formulation includes defining the decision epochs and periods, the states and action sets, the rewards and transition probabilities, the decision rules, and the policies.

A finite-horizon model where decisions are made at discrete time points is used to solve the problem. The discrete time points at which decisions are made are referred to as decision epochs. Time is divided into periods with decision epochs

representing the beginning of each period. The set of decision epochs is denoted $T \equiv \{1, 2, ..., N\}$ where $N < \infty$ indicates the time frame is of a finite-horizon. At each decision epoch, the model will be in one of a number of possible states and a decision must be made concerning the course of action to follow.

States represent all of the possible scenarios in which the the system can be observed. The set of all possible states is known as the state space. Labelling the states by the integers $s = 1, 2, ..., K$, the state space is denoted as $S \equiv \{s_1, s_2, ..., s_K\}$. At each decision epoch in which the system is found to be in state s, a decision maker must choose an action a, where actions are labelled by the integers $a = 1, 2, ..., L$ from the set of actions that are available while the system in state s. The set of actions available while in state s is denote as A_s. The number of actions L available in any state s is dependent on that state s and need not be the same for all s. The set of actions available in state s is denoted as $A_s \equiv \{a_{s,1}, a_{s,2}, \ldots, a_{s,L}\}$ where $a_{s,m}$ is the m^{th} possible action while in state s. For each action chosen by the decision maker, there is a corresponding reward (or cost) for making that decision.

In general, the reward a decision maker achieves for choosing action $a \in A_s$ in state s at decision epoch t is denoted as $r_t(s, a)$. In the case where the rewards remain the same throughout all time periods, the notation can be shortened to $r(s, a)$. Rewards are real-valued and may be positive or negative. They may be considered as income when positive and as cost when negative. In finite-horizon Markov decision process models, no decision is made at the final decision epoch N because a decision made at decision epoch N would not be implemented as decision epoch N marks the end of the time horizon being evaluated. The reward at the final decision epoch is a function of only the state and is denoted $r_N(s_N)$. When analyzing a finite-horizon model, the final decision epoch N works to summarize the results of the previous decision. The final reward, $r_N(s)$, is sometimes referred to as the salvage value because this is the value of the final state of the system at the end of the time frame being evaluated. Prior to the final decision epoch, the state of the

system at the next decision epoch is based on the current state of the system, the decision made by the decision maker, and the transition probabilities.

A transition probability is the probability that the system moves from state s to another specified state at the time of the next decision epoch given action $a \in A_s$ was chosen. Transition probabilities are denoted as $p_t(\cdot|s,a)$ where $(\cdot)$ represents the state to which the system transitions, given the system is in state s and action a is chosen. In the case where the transition probabilities remain the same throughout all time periods, the notation can be shortened to $p(\cdot|s,a)$. For models presented here, it is assumed that

$$\sum_{j \in S} p_t(j|s,a) = 1, \tag{3.1}$$

although Puterman [35], page 20, does discuss models where this equality is not required.

Decision rules define the procedure used to select the actions for each state at each decision epoch. Decision rules can be either deterministic or stochastic and either Markovian or non-Markovian [35], page 21. Of the different combinations of the characteristics, history-dependent randomized policies Π^{HR} are the most general [35], page 21. When decision rules are deterministic, actions are selected with certainty. When decision rules are randomized an action is selected randomly, according to a specified probability distribution, from the set of available actions for that state. Markovian decision rules rely only on the current state of the system. This follows from the Markov property that states the probabilistic behavior of the future of the process depends only on the current state of the process [26], page 16. In this sense, the history of the process is contained in the current state of the process. History-dependent decision rules depend on the past history of the system as represented by all of the previous states and actions.

In this work, decision rules will be deterministic and Markovian. Markovian deterministic policies Π^{MD} are a subset of history-dependent randomized policies

[35], page 22. Puterman, [35], page 89, shows that deterministic decision rules lead to optimal policies and therefore randomized decision rules will not be considered here. Deterministic Markovian decision rules are functions, $d_t(s) : S \rightarrow A_s$, where $s \in S$, $d_t(s) \in A_s$, which determine the the action chosen whenever the system is in state s at decision epoch t.

Defining a policy is the final step in formulating a Markov decision process. A policy π is a sequence of the decision rules used, $\pi = (d_1, d_2, ...d_{N-1})$. A policy that uses the same decision rule for all decision epochs, $d_t = d$, $\forall\ t$ is called a stationary policy.

In summary, Markov decision processes are composed of decision epochs and periods, states and action sets, rewards and transition probabilities, decision rules, and policies. The models presented herein are finite-horizon models, i.e., there is a finite number of discrete decision epochs. The decision epochs occur at the beginning of each period. States represent the different conditions in which the system can be observed. For each state that the system can assume, there is a set of actions from which a decision maker can choose. These actions, along with the transition probabilities, determine which state the system will be in at the next decision epoch. There is a reward associated with the selection of each action. A reward can be interpreted as either income or cost and results from the selection of a particular action. Decision rules specify how that action can be chosen and policies specify which action is chosen throughout the time horizon under consideration. The decision epochs, states, action sets, rewards, and transition probabilities make up a Markov decision process. The combination of these components combined and an optimality criterion is known as a *Markov decision problem.*

3.1.2 Optimality Criteria

In order to determine an optimal policy for a Markov decision process, there must be a means by which to compare policies to determine an ordering. Markov

decision process models are stochastic models, therefore the policies (e.g. $\pi = (d_1, d_2, ...d_{N-1})$) are vectors of random variables. Because policies are vectors of random variables, stochastic ordering is required of these vectors.

The following notation will aid in the discussion of stochastic ordering and policy comparison. Let R_t denote the random reward received in time period t when $t < N$ and let R_N denote the reward of the final decision epoch or the salvage value [35], page 74. Normally, for $t < N$ the rewards are independent of the time period and the subscript t is dropped for stationary rewards. $\mathbb{R}$ denotes the set of all real numbers, and $\mathbb{R}^n$ denotes all n-dimensional vectors of real values. The vector $R \equiv (R_1, \ldots, R_N) \in \mathbb{R}^n$ denotes a random sequence of rewards. Finally, $\Re$ denotes the set of all possible reward sequences.

For a pair of random variables it is said that the random variable U is stochastically greater than the random variable V if

$$P\{V > t\} \leq P\{U > t\}, \quad \forall \quad t \in \mathbb{R}. \tag{3.2}$$

A random vector $U = (U_1, \ldots, U_n)$ is stochastically greater than a random vector $V = (V_1, \ldots, V_n)$ if

$$E[f(V_1, \ldots, V_N)] \leq E[f(U_1, \ldots, U_N)], \quad \forall \ f : \mathbb{R}^N \to \mathbb{R} \tag{3.3}$$

where the expectation is finite and the partial ordering on $\mathbb{R}^n$ is maintained, such that $v_i \leq u_i$ for $i = 1, \ldots, N$ and $f(v_1, \ldots, v_N) \leq f(u_1, \ldots, u_N)$ [35], page 75.

According to Puterman [35], page 77, when using stochastic ordering to compare two policies π and ν the inequality

$$E^{\pi}[f(R_1, \ldots, R_N)] \geq E^{\nu}[f(R_1, \ldots, R_N)] \tag{3.4}$$

must hold for a large class of functions f which may not be representative of the decision maker's tolerance for risk. The issue of risk is raised because a decision chosen, based solely on maximizing the expected value, may seem more risky to a decision maker when compared to another decision with a lower expected value. For example, Clemens and Reilly [12], page 529, present a game similar to the following. Suppose there are two possible decisions, decision A and decision B. Each decision has two possible outcomes with each outcome having a probability of 0.5. The possible outcomes of decision A are gains of \$1 and \$100, which results in an expected gain of \$50.50. The possible outcomes of decision B are gains of \$40 and \$60, which results in an expected gain of \$50. If maximizing the expected value was the only concern decision A should be chosen. However, some people would think decision A is riskier than decision B because choosing decision B guarantees a gain of at least \$40 with a chance for more, but decision A only guarantees a gain of \$1. Therefore, it may be desirable to consider risk along with expected value when evaluating decisions. According to Bertsekas [8], pages 4 and 8, *expected utility theory* can be used to apply mathematical methods for analyzing decision problems when the decision maker is able to rank order, by preference, the probability distribution of each possible outcome.

Puterman [35], page 77, states that requiring Equation (3.4) to only hold for a specified function allows utility theory to provide a useful means of policy comparison. A utility function, $\Psi(\cdot)$, is a real-valued function representing a decision maker's preference for elements in a set W. If the decision maker prefers v over w this implies $\Psi(v) \geq \Psi(w)$. Also, if $\Psi(v) \geq \Psi(w)$, this implies that the decision maker prefers v over w. Thus, this is an if-and-only-if relationship. If the decision maker has no preference between v and w then $\Psi(v) = \Psi(w)$. Using utility $\Psi(\cdot)$ all of the elements in set W can be compared. Describing techniques for determining utility functions is beyond the scope of this work. The interested reader is referred to Fishburn [16] and Keeney and Raiffa [25].

If the elements of W are allowed to represent the outcomes of a random process then, according to Puterman [35], page 77, the "expected utility provides a total ordering on equivalence classes of outcomes." The expected utility for a discrete random variable Y is given by

$$E(\Psi(Y)) = \sum_{y \in W} \Psi(y) P\{Y = y\}. \tag{3.5}$$

Let $(\rho_1, \ldots, \rho_N)$ represent a realization of the reward process and let $P^{\pi}_{\Re}$ denote the probability distribution on the set of rewards. Then for finite-horizon Markov decision process models with discrete state spaces the expected utility of a policy π can be represented as

$$E^{\pi}[\Psi(R)] = \sum_{(\rho_1, \ldots, \rho_N) \in \Re} \Psi(\rho_1, \ldots, \rho_N) P^{\pi}_{\Re}\{(\rho_1, \ldots, \rho_N)\}. \tag{3.6}$$

When using expected utility it is clear the decision maker prefers policy π to policy ν if

$$E^{\pi}[\Psi(R_1, \ldots, R_N)] > E^{\nu}[\Psi(R_1, \ldots, R_N)]. \tag{3.7}$$

The models presented herein assume linear additive utility which is given by

$$\Psi(\rho_1, \ldots, \rho_N) = \sum_{i=1}^{N} \rho_i. \tag{3.8}$$

Linear additive utility is used because as pointed out by Puterman [35], pages 77 and 78, such utilities represent the preferences of a decision maker that is risk neutral and indifferent to the timing with which the rewards are received.

Let $v^{\pi}_N(s)$ be the expected total reward over the decision-making horizon when policy π is used and the system begins in state s. Then the expected total reward

for a deterministic Markov policy is given by

$$v_N^{\pi}(s) \equiv E_s^{\pi}\left\{\sum_{t=1}^{N-1} r_t(s_t, d_t(s_t)) + r_N(s_N)\right\}. \tag{3.9}$$

Now that the expected total reward has been defined, the optimal policy, based on that expected total reward, is established. For a model where reward is maximized, the policy with the largest expected total reward is desired and is denoted as π^*. This optimal policy π^* is found when (cf. [35], page 79)

$$v_N^{\pi^*}(s) \geq v_N^{\pi}(s), \quad s \in S$$

for all policies $\pi \in \Pi^{HR}$.

The value of a Markov decision problem, v_N^*, for the maximization case, where *sup* represents the supremum and *max* represents the maximum, is given by

$$v_N^*(s) \equiv \sup_{\pi \in \Pi^{HR}} v_N^{\pi}(s), \quad s \in S, \tag{3.10}$$

or

$$v_N^*(s) \equiv \max_{\pi \in \Pi^{HR}} v_N^{\pi}(s), \quad s \in S, \tag{3.11}$$

when the value of the supremum is attained in Equation (3.10), such as when each A_{s_t} is finite [35], page 79. The logic remains the same for the minimization case with infimum replacing supremum and minimum replacing maximum.

The expected total reward of an optimal policy π^* is the same as the value of the Markov decision problem and thus satisfies

$$v_N^{\pi^*}(s) = v_N^*(s), \quad s \in S. \tag{3.12}$$

Markov decision processes are nothing more than stochastic dynamic programming problems. Dynamic programming makes use of a fundamental recursion to efficiently calculate a result. To perform this recursion, in the context of Markov decision processes, the expected total reward of a fixed policy must first be defined. Let u_t^π denote the total expected reward obtained by implementing policy π at the decision epochs $t, t+1, \ldots, N-1$ [35], page 80. For a Markovian deterministic policy, u_t^π when $t < N$ is given by

$$\begin{aligned} u_t^\pi(s_t) &= E_{s_t}^\pi \left\{ \sum_{n=t}^{N-1} r_n(s_n, d_n(s_n)) + r_N(s_N) \right\} && (3.13) \\ &= r_t(s_t, d_t(s_t)) + \sum_{j \in S} p_t(j|s_t, d_t(s_t)) u_{t+1}^\pi(j). && (3.14) \end{aligned}$$

The difference between $u_t^\pi(s)$ and $v_N^\pi(s)$ defined above in Equation (3.9) is that $u_t^\pi(s)$ only includes rewards from decision epoch t forward, whereas $v_N^\pi(s)$ includes rewards for the entire future [35], page 80.

3.1.3 Solution Methods

The expected total reward v_N^π can be computed by inductively evaluating u_t^π using Puterman's [35], page 80, Finite Horizon Policy Evaluation Algorithm.

Finite Horizon Policy Evaluation Algorithm (for fixed $\pi \in \Pi^{MD}$)

1. Set $t = N$ and $u_N^\pi(s_N) = r_N(s_N)$.
2. If $t = 1$, stop, otherwise goto Step 3.
3. Substitute $t-1$ for t and compute $u_t^\pi(s_t)$ by

$$u_t^\pi(s_t) = r_t(s_t, d_t(s_t)) + \sum_{j \in S} p_t(j|s_t, d_t(s_t)) u_{t+1}^\pi(j). \qquad (3.15)$$

4. Return to Step 2.

Optimality equations provide a basis for determining optimal policies and for the maximization case are given by

$$u_t(s_t) = \sup_{a \in A_{s_t}} \left\{ r_t(s_t, a) + \sum_{j \in S} p_t(j|s_t, a) u_{t+1}(s_t, a, j) \right\} \tag{3.16}$$

for $t = 1, \ldots, N-1$, and the boundary condition

$$u_N(s_N) = r_N(s_N) \tag{3.17}$$

when $t = N$. When the supremum in Equation (3.16) is attained, the supremum operation can be replace with the maximum as follows

$$u_t(s_t) = \max_{a \in A_{s_t}} \left\{ r_t(s_t, a) + \sum_{j \in S} p_t(j|s_t, a) u_{t+1}(s_t, a, j) \right\}. \tag{3.18}$$

The optimality equations reduce to the policy evaluation equations, Equation (3.14) or Equation (3.15), when the supremum of all of the actions in state s_t is replaced by the action specified by the policy being evaluated.

According to Puterman, [35] page 84, the optimality equations are fundamental to Markov decision theory because of the following properties:

Property A: The solutions to the optimality equations provide the optimal return values from period t to N.

Property B: The optimality equations determine if a policy is optimal. If the expected total reward of policy π for periods t onward satisfy the optimality equations for $t = 1, \ldots, N$, then the policy is optimal.

Property C: The optimality equations provide an efficient procedure for determining optimal return functions and policies.

Property D: The optimality equations can be used to determine structural properties of optimal return functions and policies.

The above properties are very important to the study of Markov decision processes. A proof of the properties is beyond the scope of this thesis, although the interested reader is directed to Puterman [35], page 84.

Puterman [35], page 92, presents backward induction as an efficient method for solving finite-horizon discrete-time Markov decision process problems. Puterman [35], page 92, also states that for stochastic problems the enumeration and evaluation of all policies is the only way to find the solution. The Backward Induction Algorithm presented below generalizes the policy evaluation algorithm.

Backward Induction Algorithm

1. Set $t = N$ and $u_N^*(s_N) = r_N(s_N) \forall\, s_N \in S$.
2. Substitute $t-1$ for t and compute $u_t^*(s_t)$ for each $s_t \in S$ by

$$u_t^*(s_t) = \max_{a \in A_{s_t}} \left\{ r_t(s_t, a) + \sum_{j \in S} p_t(j|s_t, a) u_{t+1}^*(j) \right\}. \tag{3.19}$$

   Set

$$A_{s_t,t}^* = \arg\max_{a \in A_{s_t}} \left\{ r_t(s_t, a) + \sum_{j \in S} p_t(j|s_t, a) u_{t+1}^*(j) \right\} \tag{3.20}$$

   where $\arg\max_{a \in A_{s_t}}$ returns the set of actions (e.g. $\{a_{1,1},\ a_{1,3}\}$) which attain the maximum value of the elements evaluated.

3. If $t = 1$, stop. Otherwise return to Step 2.

The Backward Induction Algorithm is employed to find the optimal policies for the models presented in this thesis.

For completeness, it should be noted that it is also possible to formulate the problem as a linear program. Derman and Klein [14], Ross [36], and White [39] all discuss linear programming formulations for finite-horizon Markov decision pro-

cesses. The following formulations are based on the treatments of linear programming formulations given by White [39], pages 113-114, and Ross [36], pages 40-42.

The problem of finding the maximum expected total reward can be formulated as

$$\min_u \left\{ \lambda u = \sum_{s \in S} \lambda_s u_1(s) \right\} \tag{3.21}$$

subject to

$$\begin{aligned} u_t(s) \quad &\geq \quad \max_{a \in A_{s_t}} \left\{ r_t(s_t, a) + \sum_{j \in S} p_t(j|s_t, a) u_{t+1}(j) \right\}, \\ &1 \leq t \leq N-1, \forall\, s \in S \end{aligned} \tag{3.22}$$

$$u_N(s) = 0, \quad t = N, \; \forall\, s \in S \tag{3.23}$$

where λ is a vector whose length is the number of states in the system. The elements of λ represent the probability of the system beginning in state $s \in S$. The decision variables $u_t, t = 1, 2, \ldots, N$ are unrestricted in sign. Recall from Equation (3.14) that $u_t(s)$ represents the total expected reward through time periods $t, t+1, \ldots, N-1$. Equations (3.22) and (3.23) correspond to the optimality equations and boundary condition, respectively.

The formulation can be equivalently stated as

$$\min_u \left\{ \lambda u = \sum_{s \in S} \lambda_s u_1(s) \right\} \tag{3.24}$$

subject to

$$\begin{aligned} u_t(s) \quad &\geq \quad r_t(s_t, a) + \sum_{j \in S} p_t(j|s_t, a) u_{t+1}(j), \\ &1 \leq t \leq N-1, \; \forall s \; \in S, \forall\, a \in A \end{aligned} \tag{3.25}$$

$$u_N(s) = 0, \quad t = N, \forall\, s \in S \tag{3.26}$$

In some cases, such as the problem solved by this research, where expected total cost is being minimized the problem can be formulated with the objective function written as follows:

$$\min_u \left\{ \sum_{t=1}^{N} \sum_{s \in S} u_t(s) \right\}. \tag{3.27}$$

In these cases, the value of the objective function is ignored and solution is found by looking at the value of the decision variables.

By using the linear programming formulations, the optimal (minimum) costs at each decision epoch may be found. These values are the same as those found when solving the problem using Markov decision processes. Unfortunately, solving the problem via linear programming does not provide the *policy* that must be followed to obtain these optimal values. For this reason it is desirable to use Markov decision processes to solve the problem. Being able to find solutions to the problem via linear programming provides a useful check on the results of the stochastic dynamic programming solution. In addition, the linear programming formulation allows for sensitivity analysis to be extended to an analysis of the reward (cost) coefficients of the problem.

3.2 Assumptions for the Satellite Constellation Models

Various assumptions regarding satellites and their operation are employed in order to model the system as a Markov decision process, to prevent state space explosion of the models, and to produce tractable solutions. Chapter 5 discusses the relaxation of these assumptions in future work; however, in these initial models the assumptions assist in focusing attention on the solution method rather than attempting to provide a perfectly realistic model. The following two assumptions are required to model satellite constellations as presented in Sections 3.3 and 3.4.

- Satellite are assumed to be independently subject to failure. This implies that such events as solar proton events and geomagnetic storms do not affect the failure of the satellites. Satellite failures are rarely caused by such events, therefore this assumption can be considered to have minimum impact.
- The assumption of exponentially distributed satellite lifetimes greatly simplifies the modeling of satellite constellations. Exponential lifetimes enable the computation of conditional probabilities involving the interval reliability of the satellites. Barlow and Proschan [6], page 18, suggest that it is valid to assume exponentially distributed lifetimes for complex systems with many critical, independently operating components as the number of components and time in operation increases. Given the complexity and criticality of subsystems it assumed reasonable to employ this assumption.

In an effort to present the methodology and analysis in a clear and understandable manner, the following additional assumptions were made. Though the following are not rigid, they are employed in order to more easily demonstrate the analysis of the model's output.

- For the models presented herein, it is assumed that when a spare satellite is ordered, it will be available during the next time period. In reality, it can sometimes take many months to build a satellite. Molnau, Olivieri, and Spalt [31] state that for traditional space vehicle manufacturing it can typically take 18 months to build a satellite. For modern manufacturing processes, the time between producing multiple satellites can be reduced to two months. This assumption simplifies the modeling of satellite constellations. Because satellite lifetimes are relatively long in relation to the time it takes to build a new satellite and the high reliability of launch boosters the likelihood that a new satellite can be built prior to a recently replaced satellite needing replacement is high. For example, many satellites currently have lifetimes of ten years or more, but satellite production times are typically much shorter than this time

period. The main effect of allowing satellite production to exceed three months is that orders for replacement satellites would need to be place earlier.

- The models also assume that there will be at most one spare satellite for each satellite considered an active part of the constellation. There could actually be any number of spare satellites from none to multiple spare satellites per active satellite. For example, a Lockheed Martin press release [37] states that in January of 2001 there were 14 GPS satellites in storage. Allowing one spare per active satellite should allow ample spare satellites to be kept on hand. It should also provide a reasonable modeling bound for determining the state space where the number of spare satellites is a factor in state definitions.

- Satellites are assumed to be either operational or non-operational. Most satellites suffer from some degradation due the stresses of launch and the harsh space environment. Yet these degraded satellites are still capable of performing some necessary functions required by the mission. For the purpose of this study, these satellites, even though degraded, are considered operational until the degradation reaches the point at which the mission under evaluation can no longer be performed at a satisfactory level. When a satellite reaches this level of degradation, for the purpose of this model and the mission being evaluated it is considered non-operational. Secondary payloads or missions need to be evaluated separately.

- While performing policy evaluation, the models rely on the assumption that the costs involved are known with certainty. It is reasonable that the cost of building a satellite, the cost of storing a satellite, and the cost of launching a satellite are fairly well known. Conversely, the penalty cost which is assessed when the constellation is not fully operational is not as obvious. There exist a directly proportional relationship between the penalty cost and the operational level of the satellite constellation. As the penalty cost rises, so will the operational level of the satellite constellation. This relationship exists because

as the penalty cost rises the penalty charges can be avoided by making more frequent satellite replacements in order to prevent outages. Buffa and Miller [9] pages 142-143, present the idea of service levels wherein penalty cost, $c_{penalty}$, can be expressed as some function of the service level, l_s where

$$c_{penalty} = f(l_s). \tag{3.28}$$

Here f is a utility function as described in Section 3.1.2. It is beyond the scope of this work to determine the utility function for the penalty cost. A nominal value for the penalty cost is chosen in Chapter 4 and sensitivity analysis is performed on that value.

- For the purpose of this research the models assume that the cost of each satellite is constant throughout the time- horizon being evaluated. The purpose of this assumption is to focus attention on the methodology and to make analysis of the model outputs more straightforward. Often a contract will be made to build several satellites over a period of time. These contracts allow the buyer to obtain lower prices for each individual satellite. For example, *Jane's Space Directory* [5], page 573, reports that for GPS Navstar Block 2R satellites there was a design and development cost of 119 million dollars and that the first 20 satellites after that would cost a total of 575 million dollars. The total cost of development and production for the first 20 satellites is 694 million dollars. The average cost per satellite is 34.7 million dollars. Such long term contracts lock in the cost of satellites and allow the average price to be used as a constant cost for modeling.

- Each satellite is assumed to be launched separately, although in some cases multiple satellites could be launched from the same booster. This is especially evident when launching multiple satellites into the same orbital plane. For example, Iridium satellites had two satellites per launch on Long March 2

rockets, five satellites per launch on Delta 2 rockets, and seven satellites per launch on Proton K/DM rockets [22] pages 223, 121, and 292. Assuming only one satellite per launch is acceptable because this is the most frequent case. It also provides a conservative estimate of the cost. With the exception of smaller, lighter weight satellites such as Iridium and Globalstar, most satellites are launched individually [22], pages 65-66, 107-110, 196-200, 222-223, 288-292, and 364-376.

- The launch costs per satellite are also assumed to be constant. Again, the main reason to hold launch cost throughout the time horizon under evaluation is to focus attention on the methodology and make analysis of model output easier to interpret. This is assumed because each satellite is assumed to be launched separately. If multiple satellites were allowed to be launched together, then a different booster may be required that has different costs. For example, Long March 2, Delta 2 and Proton rockets were among the launchers used to launch Iridium satellites [22], pages 223, 109, and 292. Delta 2 launches boosted five Iridium satellites per launch into orbit at a cost of 50 to 60 million dollars [22], page 98. Proton launches put seven Iridium satellites per launch into orbit at a cost of 90 to 100 million dollars, [22] page 284. Depending on the number of satellites launched at the same time and booster required many different launch cost could be possible for the satellites. By considering each satellite to launched separately the launch cost can be assumed constant and system modeling is simplified. Again, this would be a conservative estimate of costs.

- On-orbit spare satellites are not considered in the models. Many constellations do not make use of on-orbit spares due to the added cost and the extra wear to the satellite caused by additional exposure to the space environment. For this reason and to keep the state space of the problem small, on-orbit spares are not considered.

- The models assume that launch facilities and launchers are available for all launches so that launches occur at the prescribed times. The Delta 2 rocket has demonstrated a peak launch rate of 12 launches per year and it is estimated that the Delta 2 has a maximum surge launch rate of 15 launches per year [22], page 99. This type of limitation exist for all launchers, for example, the Proton rocket is currently being produced at a maximum rate of 15 per year [22], page 285. The relatively long satellite lifetimes make it unlikely that large numbers of launches will be needed each year, under normal circumstances, to maintain a constellation.

- The models (as presented) do not assume any budgetary constraints. For the purpose of finding the optimal policy for maintaining the satellite constellation with minimum expected total cost this is a fair assumption. The optimal policy provides the minimum expected total cost over the time horizon evaluated. A policy derived under such an assumption is useful in creating a budget or requesting funds for the maintenance of the constellation. Having derived this optimal replacement policy places strong impetus on following the policy because following any other course of action will have a higher minimum expected total cost.

  However, if the decision is made to limit the funding for maintaining a satellite constellation during any time period, it is easy to implement budgetary constraints. By implementing budgetary constraints, the new policy will result in a minimum expected cost that is greater than or equal to the unconstrained model. To implement the budgetary constraints the actions that result in an immediate monetary reward (cost) exceeding the allowable budgeted amount can be ignored as if these actions did not exist. In this way, an optimal policy can be found that minimizes the total expected cost of maintaining the satellite constellation while still satisfying budgetary constraints.

- This model only takes into account the cost directly associated the building, storage, and launch of a satellite, as well as the opportunity cost of a satellite failure. This model does not include the costs of maintaining a ground system with which to operate the satellite.

3.3 Single-unit Model

This section presents the formulation for a model with a single satellite. In this case, the number of satellites in the constellation is $M = 1$. As presented in Section 3.1.1, it is necessary to define decision epochs, periods, states, action sets, rewards, transition probabilities, decision rules, and policies.

The single satellite model is a finite time horizon model such that $T = \{1, 2, ..., N\}$ where $N < \infty$. The states of the single satellite model represent the states of the stochastic process $\{(X_t, S_t) : t \in T\}$ where X_t represents the whether the satellite is operational and S_n represents if a spare satellite is available. For example, $(X_5 = 1, S_5 = 0)$ means that at $t = 5$ the satellite is operational ($X_5 = 1$) and there is not a spare satellite available ($S_5 = 0$). The state space for the model contains four states and is denoted as $S = \{s_1, s_2, s_3, s_4\}$. The states are defined in Table 3.1.

Table 3.1 Single satellite model state definitions.

State	Definition
s_1	the satellite is working and no replacement satellite is available
s_2	the satellite is working and a replacement satellite is available
s_3	the satellite is not working and a replacement satellite is not available
s_4	the satellite is not working and a replacement satellite is available

The set of possible actions for each state depend explicitly on the state. When the system is in state s the actions from set A_s are available for selection. The actions for the four states of this model are defined in Table 3.2.

Transition probabilities for the model are defined using the following notation:

Table 3.2 Single satellite model action definitions.

Action Set	Action	Definition
A_1	$a_{1,1}$	Let the system run without intervention
	$a_{1,2}$	Order a replacement satellite
A_2	$a_{2,1}$	Let the system run without intervention
	$a_{2,2}$	Replace the satellite with the available spare
	$a_{2,3}$	Replace the satellite and order a new replacement
A_3	$a_{3,1}$	Let the system run without intervention
	$a_{3,2}$	Order a replacement satellite
A_4	$a_{4,1}$	Let the system run without intervention
	$a_{4,2}$	Replace the satellite with the available spare
	$a_{4,3}$	Replace the satellite and order a new replacement

Interval reliability: The interval reliability of a satellite, denoted by R_{sat}, is the probability that the satellite will survive until the next decision epoch. A derivation of interval reliability is given in Proposition 3.1.

Probability of a successful launch: The probability of a successful launch, denoted as P_{sl}, is the probability that a satellite is launched and becomes operational. This probability includes the event of a successful launch into orbit, any transfer maneuvers into the final orbit, and successful completion of initial check-out procedures until the satellite is declared operational.

The transitions for this model are shown pictorially in the state transition diagram of Figure 3.2.

Proposition 3.1 derives the interval reliability when satellite lifetimes are distributed exponentially. The memoryless property of the exponential distribution is exploited in solving the problem using Markov decision processes.

Proposition 3.1 *If W is the exponentially distributed lifetime of a satellite with mean lifetime λ and τ_1 and $\tau_2 \in [0, \infty)$, where $\tau_2 = \tau_1 + \Delta t$ (Δt is the length of one*

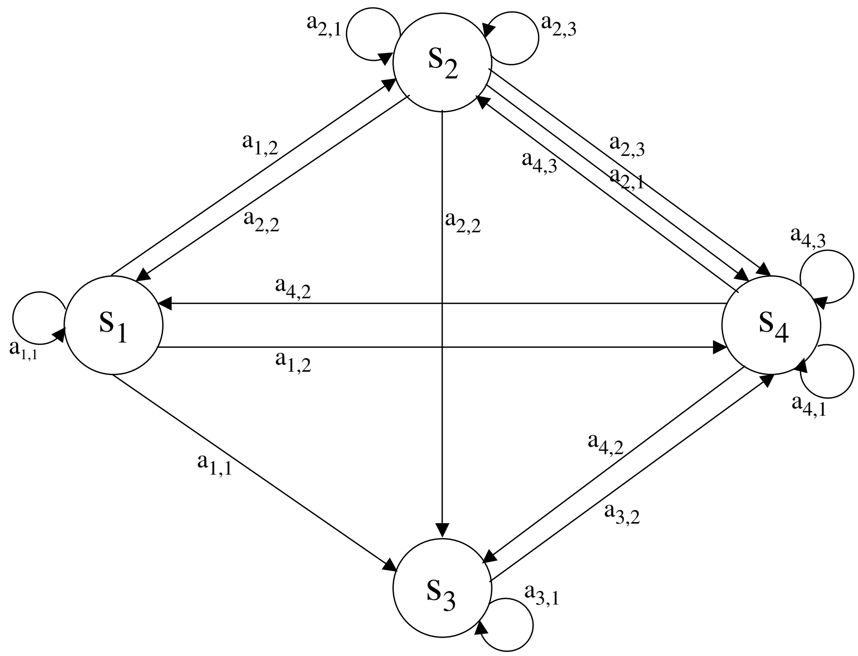

Figure 3.2 State transition diagram.

time period), then

$$P\{W > \tau_2 | W > \tau_1\} = e^{-\lambda \Delta t}, \quad \Delta t > 0. \tag{3.29}$$

Proof. The proof of Proposition 3.1 follows.

$$\begin{aligned} P\{W > \tau_2 | W > \tau_1\} &= P\{W > \tau_1 + \Delta t | W > \tau_1\} \\ &= \frac{P\{W > \tau_1 + \Delta t, W > \tau_1\}}{P\{W > \tau_1\}} \\ &= \frac{P\{W > \tau_1 + \Delta t\}}{P\{W > \tau_1\}} \\ &= \frac{(1 - (1 - e^{-\lambda(\tau_1 + \Delta t)}))}{(1 - (1 - e^{-\lambda \tau_1}))} \\ &= \frac{e^{-\lambda(\tau_1 + \Delta t)}}{e^{-\lambda \tau_1}} \\ &= e^{-\lambda \Delta t} \end{aligned}$$

■

Table 3.3 summarizes the transition probabilities for the single-unit model.

Table 3.3 Single satellite model transition probabilities.

Current State	Probability	Value
s_1	$p_t(s_1\|s_1, a_{1,1})$	R_{sat}
	$p_t(s_3\|s_1, a_{1,1})$	$1 - R_{sat}$
	$p_t(s_2\|s_1, a_{1,2})$	R_{sat}
	$p_t(s_4\|s_1, a_{1,2})$	$1 - R_{sat}$
s_2	$p_t(s_2\|s_2, a_{2,1})$	R_{sat}
	$p_t(s_4\|s_2, a_{2,1})$	$1 - R_{sat}$
	$p_t(s_1\|s_2, a_{2,2})$	P_{sl}
	$p_t(s_3\|s_2, a_{2,2})$	$1 - P_{sl}$
	$p_t(s_2\|s_2, a_{2,3})$	$P_{sl} + R_{sat} \times (1 - P_{sl})$
	$p_t(s_4\|s_2, a_{2,3})$	$(1 - R_{sat}) \times (1 - P_{sl})$
s_3	$p_t(s_3\|s_3, a_{3,1})$	1
	$p_t(s_4\|s_3, a_{3,2})$	1
s_4	$p_t(s_4\|s_4, a_{4,1})$	1
	$p_t(s_1\|s_4, a_{4,2})$	P_{sl}
	$p_t(s_3\|s_4, a_{4,2})$	$1 - P_{sl}$
	$p_t(s_2\|s_4, a_{4,3})$	P_{sl}
	$p_t(s_4\|s_4, a_{4,3})$	$1 - P_{sl}$

In this model all of the rewards are actually costs. The costs used here are defined as follows:

Satellite Costs: Satellite costs, denoted as c_{sat}, are all of the costs involved in purchasing a new satellite.

Holding Costs: Holding costs, denoted as c_{hold}, are the costs associated with storing a satellite prior to its launch.

Launch Costs: Launch costs, denoted as c_{launch}, include all of the costs associated with the launching of a satellite. These costs include the launch booster, the shipment of the launch booster, launch range support, and etc.

Penalty Costs: Penalty cost, denoted as $c_{penalty}$, are the costs associated with not keeping each satellite in the constellation at an operational level.

The rewards for the single satellite model are shown in Table 3.4.

Table 3.4 Single satellite model rewards.

Current State	Reward	Definition
s_1	$r_t(s_1, a_{1,1})$	0
	$r_t(s_1, a_{1,2})$	c_{sat}
s_2	$r_t(s_2, a_{2,1})$	c_{hold}
	$r_t(s_2, a_{2,2})$	c_{launch}
	$r_t(s_2, a_{2,3})$	$c_{launch} + c_{sat}$
s_3	$r_t(s_3, a_{3,1})$	$c_{penalty}$
	$r_t(s_3, a_{3,2})$	$c_{sat} + c_{penalty}$
s_4	$r_t(s_4, a_{4,1})$	$c_{penalty} + c_{hold}$
	$r_t(s_4, a_{4,2})$	$c_{launch} + c_{penalty}$
	$r_t(s_4, a_{4,3})$	$c_{launch} + c_{sat} + c_{penalty}$

The minimum expected total cost $u_t^*(\cdot)$ is defined for state s_t by

$$u_t^*(s_t) = \max_{a \in A_{s_t}} \left\{ \sum_{j \in S} p_t(j|s_t, a) u_{t+1}^*(s_t, a, j) + r_t(s_t, a) \right\} \tag{3.30}$$

where the cost are defined with negative values and the boundary value

$$u_N(s_N) = r_N(s_N). \tag{3.31}$$

3.4 Multi-unit Model

This section presents the formulation of the multi-unit model. A system of multiple satellites is by definition a constellation. Here M, the number of satellites in the system, is two or more, $(M \geq 2)$. As presented in Section 3.1.1, it is necessary to define decision epochs, periods, states, action sets, rewards, transition probabilities, decision rules, and policies. This model has a finite time horizon with the decision epochs occurring at the beginning of each time period, so that $T = \{1, 2, ..., N\}$ where $N < \infty$.

The states of the model can be defined by making use of following observations. Each state is made up of two pieces of information; the satellites which are operational and the number of spare satellites currently available. Thus the system is a stochastic process $\{(X_t, S_t) : t \in T\}$ where X_t represents which of the satellites are operational at decision epoch t and S_t represents the number of spare satellites available at decision epoch t. For example, the case $(X_2 = 5, S_2 = 3)$ means at decision epoch $t = 2$, $X_2 = 5$ might mean for example that satellites $1, 2$, and, 4 are operational and $S_2 = 3$ means three spare satellites are available.

For any multi-unit system there is a set of cases representing each possible combination of satellites working and not working, the X_t's. This set includes cases ranging from all of the satellites working to none of the satellites working. For this model it is important to distinguish between individual satellites so that the case with satellites 1 and 2 working is distinct from the case with satellites 1 and 3 working. Each of these cases represent distinct states of the system. Each possible value of X_t is paired with each possible value of S_t (the number of spare satellites available) in order to form the states of the model. If the maximum number of spares satellites allowed in the system is the same as the number of satellites in the system, M, then there are $M + 1$ states corresponding to each value of X_n. There are $M + 1$ states because there is a state representing each possible value of S_t, $0, 1, \ldots, M$.

The total number of states in the model is clearly dependent on M. With $M \geq 2$ the total number of states in a system with M satellites can be found using the equation

$$\left\{ \sum_{j=0}^{M} {}_M C_j \right\} \times (M + 1) \tag{3.32}$$

where ${}_M C_j$ represents the number of combinations on M objects taken j at a time. Combinatorial growth of the state space is clearly present.

Actions for the model are dependent on the state of the system and are defined by applying the following rules. A set of actions is determined for each system

state by comparing the current state of the system to the system state that has M operational satellites and a full complement of M spare satellites. Actions range from maintaining the current system status to moving the system towards the most robust system state (e.g. the state with M operational satellites and M spare satellites).

The simplest and most basic action is to allow the system to continue running with intervention. The next type of action to consider is the purchase of spare satellites until there are M spares available. For example, if there are currently k satellites on hand, there are $M-k$ actions corresponding to purchase of spare satellites in the quantities $1, 2, \ldots, M-k$. The final class of actions to consider is what to do with spare satellites currently on hand. These satellites can be used to replace any of the satellites in the model. There are actions representing every possible combination of replacement using the replacement satellites that are currently available. Whether specific satellites are operational or failed is not taken into account when defining the actions. Each possible action that follow the set of rules is enumerated. The algorithm will select which actions are necessary to minimize the expected total cost.

The number of actions for a state with k replacement satellites available at the beginning of the time period is determined using the following equations:

$$M - k + 1, \quad M \geq 2, \quad k = 0, \tag{3.33}$$

and

$$\left\{ \sum_{j=1}^{k} ({}_{M}C_{j}) \times (M - k + 1 + j) \right\} + M - k + 1, \quad M \geq 2, \quad k \geq 1. \tag{3.34}$$

The transition probabilities for the multi-unit model use the same definitions presented as Section 3.3 with an extended subscript denoting which satellite is being referred to (e.g. R_{sat1} and P_{slM}). Extending the subscript for the interval reliability allows the different satellites to have different lifetimes. Being able to allow different

satellite lifetimes is useful because upgrades are often made to replacement satellites during their construction. Extending the subscript for the probability of a successful launch is also useful because launching satellites into different orbits causes different stresses during the launch. These variations can lead to different launch reliabilities.

The reliability parameters for the multi-unit problem are computed in the same manner as those of the single-unit model. However, the transition probabilities for the multi-unit problem are much more complex than those of the single-unit problem due to the possibility of multiple events leading to the transition from some state s_c to some state s_d. To compute the transition probabilities, each event that can lead to a transition from state s_c to state s_d when action $a_{c,n}$ is chosen must be determined. The transition probability, $p_t(s_d|s_c, a_{c,n})$, is found by summing the probabilities of each of these events given they are independent. For example, assume there is a constellation consisting of two satellites, A and B, both of which are operational. If the decision is made to replace satellite A there are two possible events that can take place and system still have both satellites operational at the next decision epoch. First, if there is a successful launch that replaces satellite A and satellite B continues to function the both satellites will be operational. Secondly, if the launch to replace satellite A is unsuccessful, but satellite the original satellite A continues to function, as does satellite B, then both satellites will still be considered operational. Thus, the probability of each event occurring must be determined. Because the events are independent, the probability of each event is summed to determine the transition probability from the state with both satellites operational, back to the state when both satellites are operational, when the decision is replace satellite A.

Determining the rewards for the multi-unit model is straight forward. The rewards for the multi-unit system are actually costs and use the same cost definitions as used by the single-unit model. For decisions resulting in the purchase of a satellite the cost of a satellite, c_{sat}, is assessed. This cost is assessed for each satellite purchased at that decision epoch or as a result of that action. Holding costs, c_{hold},

are assessed whenever there is a satellite available for replacement at the beginning of the time period (i.e. at the decision epoch). This charge is assessed for each satellite in holding status. Launch costs, c_{launch}, are assessed for each satellite that is launched during the time period. Penalty costs, $c_{penalty}$, are assessed for each satellite that is not operational at the beginning of the period.

The minimum expected total cost $u_t^*(\cdot)$ is defined for state s_t by

$$u_t^*(s_t) = \max_{a \in A_{s_t}} \left\{ \sum_{j \in S} p_t(j|s_t, a) u_{t+1}^*(s_t, a, j) + r_t(s_t, a) \right\} \tag{3.35}$$

where the costs are defined with negative values and the boundary value

$$u_N(s_N) = r_N(s_N). \tag{3.36}$$

This chapter presented a review of Markov decision processes. The review covered the components required to formulate a Markov decision process, optimality criteria, and solution methods for determining optimal policies. Following the review, the chapter discusses the assumptions made about satellite constellations in order to solve the problem using Markov decision processes. After addressing these concerns, models for a single-unit problem and a multi-unit problem are given. In Chapter 4, numerical examples using notional data are given for both the single-unit and multi-unit problems. Sensitivity analysis of the notional values for satellite lifetime and costs follows the examples.

4. Numerical Results and Analysis

This chapter presents numerical results and analysis of the models using notional data. The numerical results are based on the single-unit model presented in Section 3.3 and the multi-unit model presented in Section 3.4. Moreover, a sensitivity analysis of model parameters, such as the mean satellite lifetimes and the penalty costs, is provided.

4.1 Single-unit Example

A single-unit example is first provided to demonstrate the means by which to find an optimal replacement policy using Markov decision processes. The same techniques used to find a solution for the single-unit problem are then applied to a more complicated multi-unit problem.

The single-unit example uses three months (or one quarter) of the fiscal year as the time periods. The decision epochs are defined to be $T = 1, 2, \ldots, N$ where $N = 40$ quarters which represents a 10-year time horizon. The states for this model are that the single satellite is operating or failed and whether a spare replacement satellite is available. These are identical to the states given in Table 3.1 and are reproduced in Table 4.1 for convenience.

Table 4.1 Single satellite model state definitions.

State	Definition
s_1	the satellite is working and no replacement satellite is available
s_2	the satellite is working and a replacement satellite is available
s_3	the satellite is not working and a replacement satellite is not available
s_4	the satellite is not working and a replacement satellite is available

The actions for the single-unit model relate to replacing the satellite, ordering a new replacement, or allowing the system to run without intervention. The actions for the model were given in Table 3.2 and are reproduced in Table 4.2 for convenience.

Table 4.2 Single satellite model action definitions.

Action Set	Action	Definition
A_1	$a_{1,1}$	Let the system run without intervention
	$a_{1,2}$	Order a replacement satellite
A_2	$a_{2,1}$	Let the system run without intervention
	$a_{2,2}$	Replace the satellite with the available spare
	$a_{2,3}$	Replace the satellite and order a new replacement
A_3	$a_{3,1}$	Let the system run without intervention
	$a_{3,2}$	Order a replacement satellite
A_4	$a_{4,1}$	Let the system run without intervention
	$a_{4,2}$	Replace the satellite with the available spare
	$a_{4,3}$	Replace the satellite and order a new replacement

A few reliability parameters must first be determined in order to specify the transition probabilities for the model. The values chosen in this thesis are of a notional nature, but are selected to be representative of real world systems where possible. Because satellite lifetimes have been assumed to be exponentially distributed only one parameter, the mean lifetime of the satellite, must be specified. *Jane's Space Directory* [5], page 573, reports that GPS Navstar Block 2R satellites have a design life of 10 years. The GPS Block 2R design life is the notional value assumed for mean satellite lifetime in this example. The mean satellite lifetime is represented by λ^{-1} where $\lambda^{-1} = 10$ years or 40 quarters. After λ has been specified the interval reliability can be determined. As shown in Proposition 3.1, the interval reliability is given by $e^{-\lambda \Delta t}$ where Δt is the amount of time included in the interval. In this case, Δt amounts to three months or one time period, therefore $\Delta t = 1$.

The final probability that needs to be specified is the probability of a successful launch. Recall from Section 3.3 that the probability of a successful launch is defined as the probability of a satellite being launched and becoming operational. *Jane's Space Directory* [5], page 263, states that the Delta 2 rocket, the primary launch vehicle of the GPS satellite, has a vehicle success rate of 99 percent. A notional value of 95 percent is used for the probability of a successful launch. This value is determined by considering the joint event that the launch is successful and that the

satellite does not fail during the checkout phase. These reliability parameters are summarized in Table 4.3.

Table 4.3 Single satellite model reliability parameters.

Reliability	Notation	Numerical Value
Mean lifetime of the satellite	λ^{-1}	40.0000
Interval reliability of the satellite	R_{sat}	0.9753
Probability of a successful launch	P_{sl}	0.9500

Once the reliability parameters have been specified the transition probabilities can be determined. Table 4.4 lists the definitions of the transition probabilities and their values.

Table 4.4 Single satellite model transition probabilities.

Current State	Probability	Definition	Value
s_1	$p_t(s_1\|s_1, a_{1,1})$	R_{sat}	0.9753
	$p_t(s_3\|s_1, a_{1,1})$	$1 - R_{sat}$	0.0247
	$p_t(s_2\|s_1, a_{1,2})$	R_{sat}	0.9753
	$p_t(s_4\|s_1, a_{1,2})$	$1 - R_{sat}$	0.0247
s_2	$p_t(s_2\|s_2, a_{2,1})$	R_{sat}	0.9753
	$p_t(s_4\|s_2, a_{2,1})$	$1 - R_{sat}$	0.0247
	$p_t(s_1\|s_2, a_{2,2})$	P_{sl}	0.9500
	$p_t(s_3\|s_2, a_{2,2})$	$1 - P_{sl}$	0.0500
	$p_t(s_2\|s_2, a_{2,3})$	$P_{sl} + R_{sat} \times (1 - P_{sl})$	0.9988
	$p_t(s_4\|s_2, a_{2,3})$	$(1 - R_{sat}) \times (1 - P_{sl})$	0.0012
s_3	$p_t(s_3\|s_3, a_{3,1})$	1	1.0000
	$p_t(s_4\|s_3, a_{3,2})$	1	1.0000
s_4	$p_t(s_4\|s_4, a_{4,1})$	1	1.0000
	$p_t(s_1\|s_4, a_{4,2})$	P_{sl}	0.9500
	$p_t(s_3\|s_4, a_{4,2})$	$1 - P_{sl}$	0.0500
	$p_t(s_2\|s_4, a_{4,3})$	P_{sl}	0.9500
	$p_t(s_4\|s_4, a_{4,3})$	$1 - P_{sl}$	0.0500

Before the total reward for each action can be determined some basic reward values must first be specified. The cost of satellites varies greatly depending on the mission. Table 4.5 is produced from a table presented by Apgar, Bearden, and Wong [2] who list the average unit cost in fiscal year 2000 dollars for various types

of satellites. The cost of a GPS (Block 2) satellite is given to be 57 million fiscal year 2000 dollars in Table 4.5. The notional value of 50 million dollars is used as a representative value for the cost of a satellite.

Table 4.5 Satellite cost in FY00 $M (from Apgar, Bearden, and Wong.)

Mission	Satellite	Average Unit Costs (FY00 $M)
Communications	Intelsat VIII	133
	TDRSS	126
	DSCS IIIB	114
Navigation	GPS (Block 2)	57
Missile Warning	DSP	314
Weather	GOES	84
	DMSP	88

According to *Jane's Space Directory* [5], page 573, the Government Accounting Office (GAO) estimated in 1990 that it would cost $\$200,000$ *annually* for each GPS satellite kept in storage. This value is used for as the nominal value for holding cost will assumed to $\$50,000$ *per quarter*. Sensitivity analysis of this value will be studied to determine its affect on the minimum expected total cost.

As stated in Section 3.2, a Delta 2 rocket costs from 50 to 60 million dollars. The nominal value for the the cost of launch used in the example is 55 million dollars. The nominal value for the penalty cost is chosen to be 50 million dollars. Given this is a notional value, sensitivity analysis will be performed in the following section. The basic reward (cost) values are summarized in Table 4.6.

Table 4.6 Single satellite model costs.

Cost	Value
c_{sat}	$\$50,000,000$
c_{hold}	$\$50,000$
c_{launch}	$\$55,000,000$
$c_{penalty}$	$\$50,000,000$

Once the basic reward values have been specified, the reward values for each action can be determined. The total reward for each model action are given in Table 4.7.

Table 4.7 Single satellite model reward values.

Current State	Reward	Definition	Value
s_1	$r_t(s_1, a_{1,1})$	0	\$0
	$r_t(s_1, a_{1,2})$	c_{sat}	\$50, 000, 000
s_2	$r_t(s_2, a_{2,1})$	c_{hold}	\$50, 000
	$r_t(s_2, a_{2,2})$	c_{launch}	\$55, 000, 000
	$r_t(s_2, a_{2,3})$	$c_{launch} + c_{sat}$	\$105, 000, 000
s_3	$r_t(s_3, a_{3,1})$	$c_{penalty}$	\$50, 000, 000
	$r_t(s_3, a_{3,2})$	$c_{sat} + c_{penalty}$	\$100, 000, 000
s_4	$r_t(s_4, a_{4,1})$	$c_{penalty} + c_{hold}$	\$50, 050, 000
	$r_t(s_4, a_{4,2})$	$c_{penalty} + c_{launch}$	\$105, 000, 000
	$r_t(s_4, a_{4,3})$	$c_{penalty} + c_{launch} + c_{sat}$	\$155, 000, 000

After specifying all of the reliability, cost, and reward parameters the problem can be solved using the Backward Induction Algorithm presented of Section 3.1.3. The algorithm was programmed using the mathematical computing package MATLAB®. Thirty runs of the model were made to assess the run time characteristics. Relevant statistics for are listed in Table 4.8. The runs were conducted on a Dell® Inspiron 8100 with a 1 gigahertz Intel® Pentium® *III* processor, 256 megabytes of RAM, and using the Microsoft® Windows 2000 Professional operating system.

Table 4.8 Single-unit run time characteristics.

Execution Statistic	Time (seconds)
Mean	0.1142
Mode	0.1100
Minimum	0.1100
Maximum	0.1210
Standard Deviation	0.0050

The policy derived from this algorithm is shown in Table 4.9. The table lists which action should be selected, depending on what state the system is in during time epochs $1, \ldots, 40$, in order to minimize the expected total cost. For example, if the system is in state s_4, during any of the first 19 time epochs, action $A_{4,3}$ (replace the satellite and order a new replacement) should be chosen. If the system is in state s_4 during the time epochs 20 through 38, then action $A_{4,2}$ (replace the satellite with the available spare) should be chosen. If in state s_4 during time epoch 39, action $A_{4,1}$ (let the system run without intervention) should be chosen. No decisions are made in time epoch 40 as this is the last epoch of the time horizon. After a decision is made it is implemented at the beginning of the next time period. There is no time following decision epoch 40 for this example, so a decision made at time 40 would not be implemented.

Table 4.9 Single satellite policies.

Epoch	s_1	s_2	s_3	s_4	Epoch	s_1	s_2	s_3	s_4
1	2	1	2	3	21	1	1	2	2
2	2	1	2	3	22	1	1	2	2
3	2	1	2	3	23	1	1	2	2
4	2	1	2	3	24	1	1	2	2
5	2	1	2	3	25	1	1	2	2
6	2	1	2	3	26	1	1	2	2
7	2	1	2	3	27	1	1	2	2
8	2	1	2	3	28	1	1	2	2
9	2	1	2	3	29	1	1	2	2
10	2	1	2	3	30	1	1	2	2
11	2	1	2	3	31	1	1	2	2
12	2	1	2	3	32	1	1	2	2
13	2	1	2	3	33	1	1	2	2
14	2	1	2	3	34	1	1	2	2
15	2	1	2	3	35	1	1	2	2
16	2	1	2	3	36	1	1	2	2
17	2	1	2	3	37	1	1	2	2
18	2	1	2	3	38	1	1	2	2
19	1	1	2	3	39	1	1	2	1
20	1	1	2	2	40	0	0	0	0

Table 4.10 shows the minimum expected cost for the following the policy of Table 4.9. Note that the minimum expected cost is dependent upon the initial state of the system.

Table 4.10 Single satellite minimum expected cost.

Initial State	Value ($M)
s_1	243.358
s_2	188.408
s_3	553.442
s_4	403.810

4.2 Analysis of the Single-unit Example

For the notional example presented the rewards (costs) are represented as notional values. When evaluating an actual constellation the rewards (costs) are estimates of the actual values. Because the values of the rewards are approximate, it is desirable to investigate the impact of these values on the minimum expected total cost. This sensitivity analysis is performed by varying a parameter, such as the mean satellite lifetime or the cost of a launch, and plotting the minimum expected total cost for each of the parameter values evaluated. The plots for a single parameter show a curve for each possible initial system state. As in Table 4.10, the minimum expected total cost is dependent on the initial state.

The first parameter evaluated is the mean satellite lifetime. Recall from Table 4.3 that a mean satellite lifetime of 40 quarters was assumed. In Figure 4.1 the mean satellite lifetime is varied from 1 quarter (3 months) to 80 quarters (20 years). The graph shows that as the mean satellite lifetime increases, the minimum expected total cost decreases, as expected. It also shows that as the mean satellite lifetime approaches 80 quarters, the decrease in the minimum expected total cost "flattens out." For this example, if mean satellite lifetimes are significantly shorter than the 10-year design life, the minimum expected total cost would increase dramatically.

However, if the mean satellite lifetimes are longer than the 10-year design life, the minimum expected total cost will decrease, but not significantly. For example, when the mean satellite lifetime is 24 quarters (6 years) or longer, the rate of decrease of the minimum expected total cost is less than 10 million dollars for each additional quarter of satellite mean lifetime. The the mean satellite lifetime is 40 quarters (10 years - the satellite design life) the rate of decrease of the minimum expected total cost is approximately three-and-a-half million dollars for each additional quarter of satellite mean lifetime. The real-world trend has been for satellites to last for longer than their design life. Such analysis may also be useful in establishing the design reliability of a system when it is being planned.

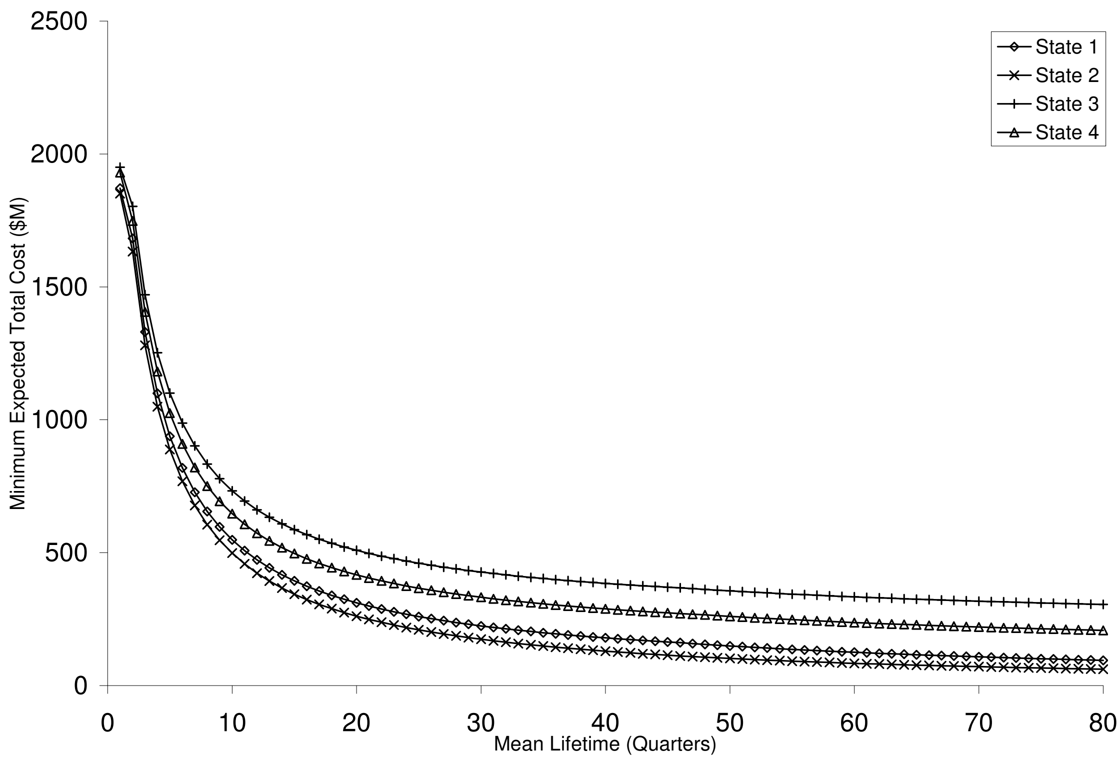

Figure 4.1 Varying the mean satellite lifetime over a 10-year time horizon.

Another parameter of interest is the penalty cost. Figure 4.2 shows how varying the penalty cost from zero to 100 million dollars per quarter affects the minimum total expected cost for a 10-year time horizon. The graph shows a very low minimum

expected total cost when the penalty cost is close to zero because a very low penalty cost implies there is a very low need or desire to maintain the satellite.

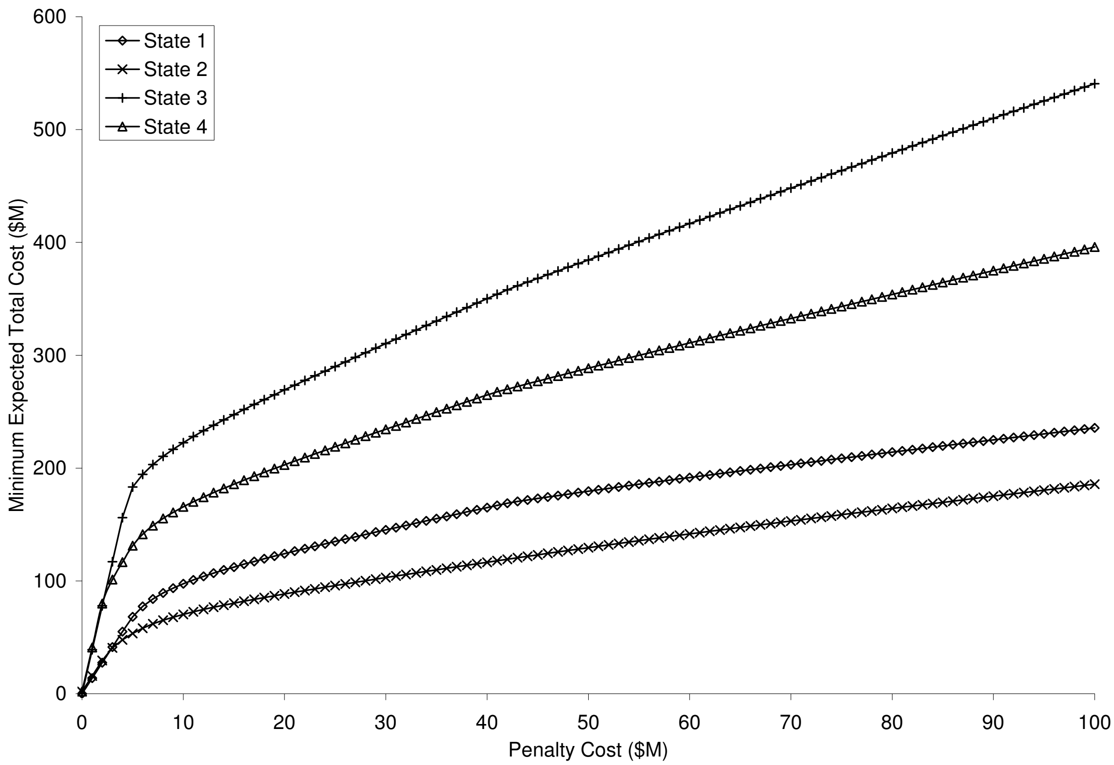

Figure 4.2 Varying the penalty cost over a 10-year time horizon.

When evaluating the case with a very low penalty cost the algorithm determines it is cheaper to pay the penalty cost than to maintain the satellite. The leftmost points of the plot in Figure 4.3 illustrate this point. This graph shows the minimum expected total cost, as well as its individual components, the expected satellite cost, the expected launch cost, the expected holding cost, and the expected penalty cost when following the optimal policy. To aide in reading the graph, these costs one are displayed for only state. State 3, the case with no operational satellites and no spares available, is chosen because this case is equivalent to populating a new constellation. When the penalty cost ranges from zero through four million dollars, the expected launch cost is zero. When the penalty cost is five million dollars, the expected launch cost jumps to 61.792 million dollars and the expected penalty

cost drops to 65.354 million dollars. With a penalty cost of four million dollars the expected penalty cost is 156 million dollars. For this example, once the penalty cost reaches a relatively small value (5 million dollars) compared to the cost of buying and launching a satellite (105 million dollars combined), the algorithm determines it is less expensive to maintain the satellite than to pay the penalty costs. When evaluating a new satellite constellation, if the penalty cost is so low that it is cheaper to pay the penalty than to maintain the constellation, then the constellation may not be needed. Such a case suggests evaluating different alternatives for achieving the mission, such as accomplishing the mission by putting a secondary payload on some other constellation.

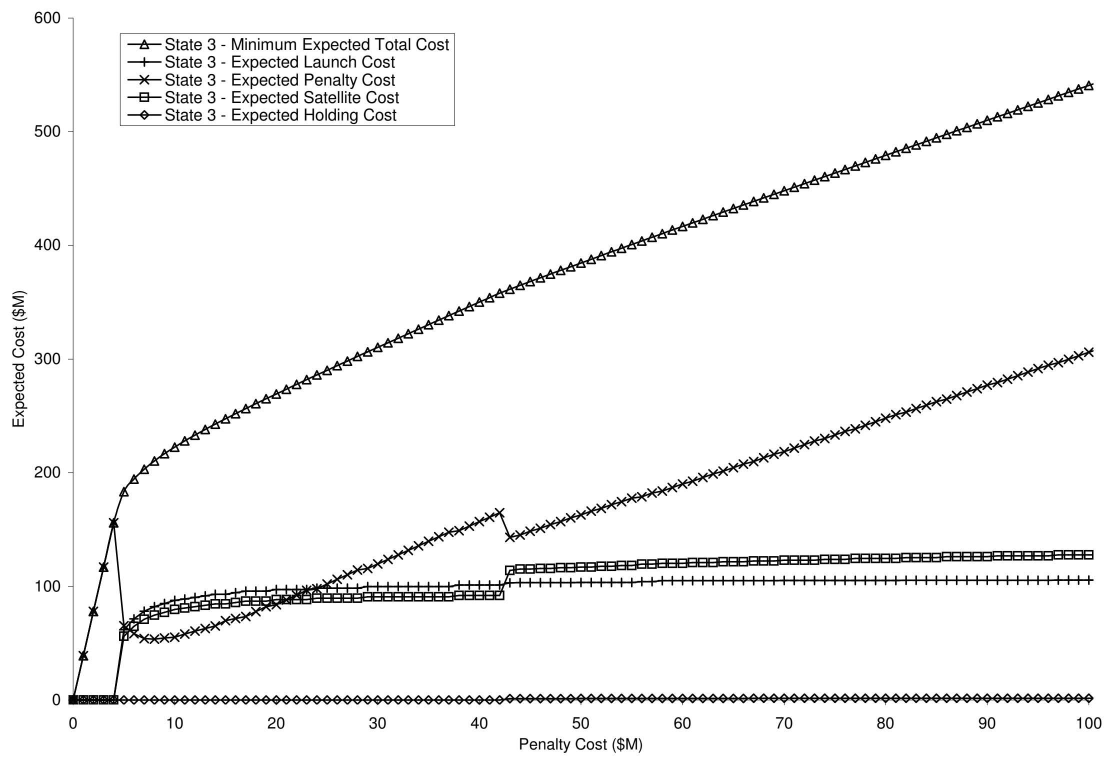

Figure 4.3 Costs over a ten year time horizon for the single-unit example.

As the the penalty cost rises the algorithm must balance the cost of maintaining the satellite and paying the penalty cost for not being able to perform the mission. The minimum expected total cost rises as the penalty cost rises for two reasons.

First, whenever a penalty must be paid the cost is higher. A second reason is that more money will be spent in buying, holding, and launching satellites in order to prevent paying the higher penalty cost than otherwise would have been spent on maintaining the satellite if the penalty cost was lower.

When the penalty cost changes from 42 to 43 million dollars the graph of Figure 4.3 shows a significant decrease in the expected penalty cost and significant increase in the expected cost of satellites. With penalty cost of 42 million dollars or less the expected holding cost is zero dollars, which implies there no satellites should be held in storage for this range of penalty cost to have the highest probability of achieving the minimum expected total cost. When the penalty cost is 43 million dollars the holding cost increase to 1.162 million dollars indicating that satellites should be held in storage to achieve the minimum expected total cost. The expected cost of satellites increases at this point because the satellite being held in storage must now be purchased. The expected penalty cost drops at this point because if the satellite fails, a replacement can be launched in the next time period. To replace a failed satellite if there is no satellite in storage, a replacement must be ordered in one time period and launched in the next.

Figure 4.4 graphs the minimum expected total cost when the launch cost is varied from zero to 150 million dollars. The graph shows that as the launch cost increase so does the minimum total expected cost of maintaining the satellite, as expected. The costs increase linearly and straight-forward to interpret. A graph of the minimum expected total cost when varying the cost of a satellite also increases linearly. Because of the uncomplicated nature of this graph it is not presented here.

The minimum total expected cost when varying the holding cost from zero to 100 million dollars is shown in Figure 4.5. The graph shows that for this example, the holding cost has almost no effect the minimum total expected cost. The holding cost is insignificant because it is small relative to the cost of purchasing and launching a satellite as well as the penalty cost for failing to maintain the satellite. By

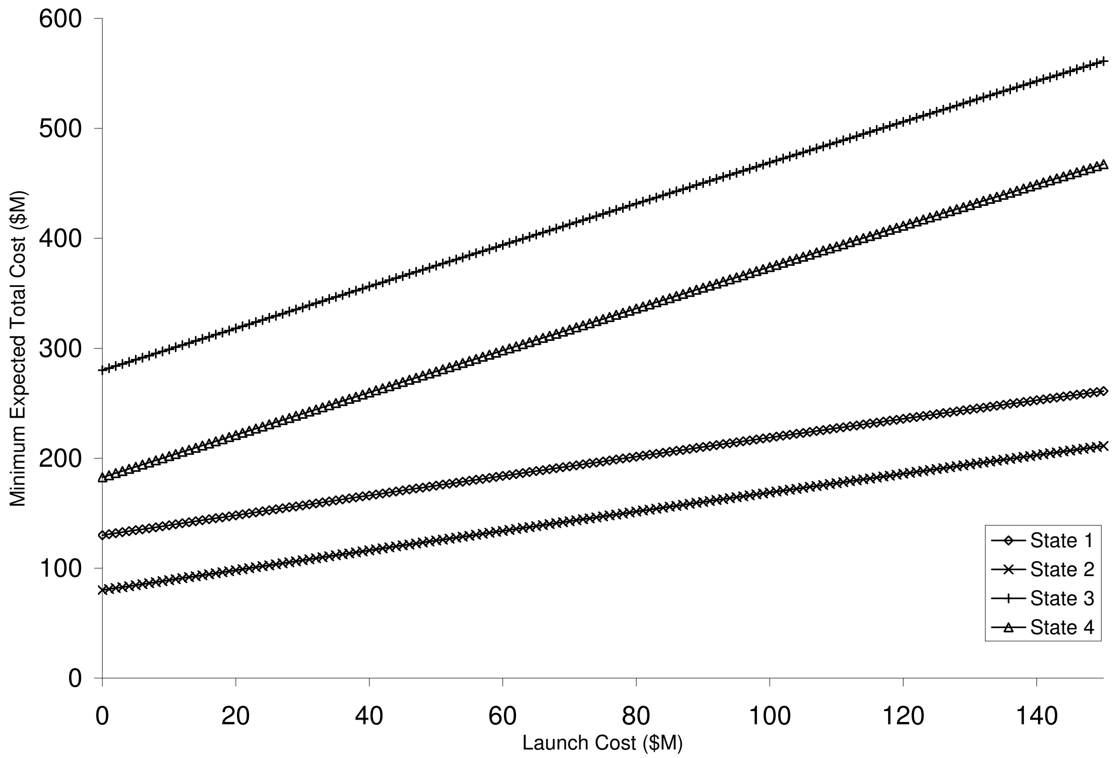

Figure 4.4 Varying the launch cost over a 10-year time horizon.

taking advantage of the ability to store satellites the penalty cost can be minimized. In this example, if the satellite failed and no replacement was available, a replacement would have to be ordered and then the replacement could be launched in the next time period. This would result in paying a penalty for two time periods. If a replacement satellite was available when the satellite failed the replacement could be launched immediately given the earlier assumption that launch vehicles and facilities are always available. In this case, a penalty is only paid for one time period. Clearly, the size of the penalty cost and the duration of the penalty period effects the minimum expected total cost. Analysis of the model allows a policy for the use of spare satellites for the constellation to be determined.

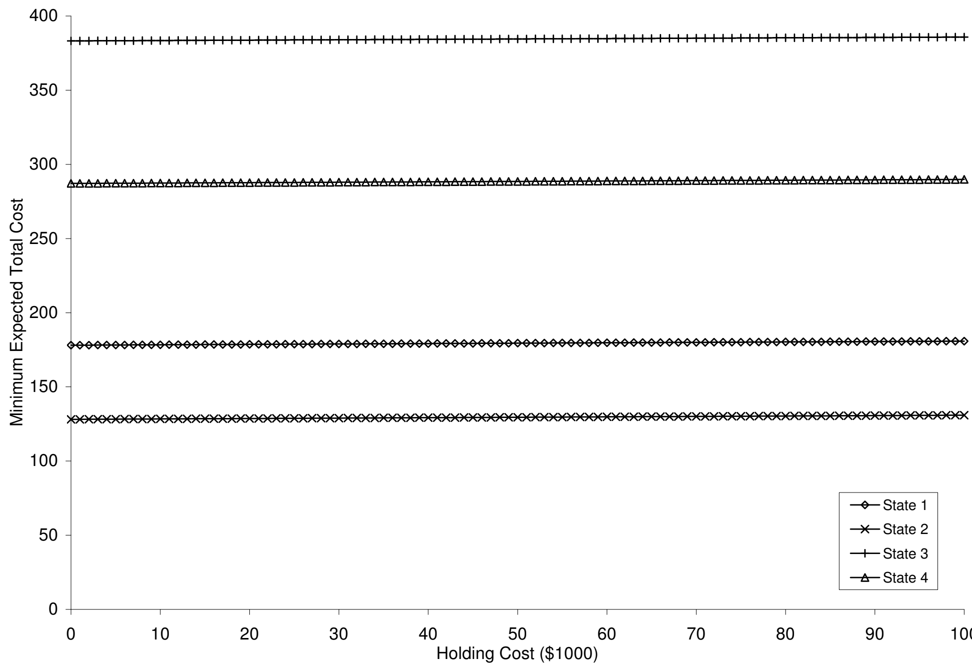

Figure 4.5 Varying the holding cost over a 10-year time horizon.

4.3 Multi-unit Example

The multi-unit example shows how the model works for a constellation of satellites. The decision epochs are defined to be $T = 1, 2, \ldots, N$ where $N = 40$ quarters which represents a 10-year time horizon. The states were described in Section 3.4 where each states specifies which satellites are operational and the number of spare satellites available. The states are listed in Table 4.11 for the case where there are $M = 3$ satellites in the constellation.

The action sets for the three satellite constellation problem are defined using the rules from Section 3.4. Following those rules, states sharing an equal number of spare satellites available also share the set of actions which can be performed in one of those states. For example, states s_1, s_5, s_9, s_{13}, s_{17}, s_{21}, s_{25}, and s_{29} each have zero spare satellites available at the beginning of the time period, and therefore each states has the same set of available actions. The actions for the three satellite

Table 4.11 Three satellite problem state definitions.

State	Working Satellites	Available Spares	State	Working Satellites	Available Spares
s_1	$1, 2, 3$	0	s_{17}	1	0
s_2	$1, 2, 3$	1	s_{18}	1	1
s_3	$1, 2, 3$	2	s_{19}	1	2
s_4	$1, 2, 3$	3	s_{20}	1	3
s_5	$1, 2$	0	s_{21}	2	0
s_6	$1, 2$	1	s_{22}	2	1
s_7	$1, 2$	2	s_{23}	2	2
s_8	$1, 2$	3	s_{24}	2	3
s_9	$1, 3$	0	s_{25}	3	0
s_{10}	$1, 3$	1	s_{26}	3	1
s_{11}	$1, 3$	2	s_{27}	3	2
s_{12}	$1, 3$	3	s_{28}	3	3
s_{13}	$2, 3$	0	s_{29}	None	0
s_{14}	$2, 3$	1	s_{30}	None	1
s_{15}	$2, 3$	2	s_{31}	None	2
s_{16}	$2, 3$	3	s_{32}	None	3

constellation are summarized in four tables (Tables **??** - **??**) representing the four levels of spare satellites available, $0, 1, 2, 3$.

Table 4.12 lists the actions applicable to states with no spare satellites available at the decision epoch. The action sets summarized in this table are A_1, A_5, A_9, A_{13}, A_{17}, A_{21}, A_{25}, and A_{29}. These action sets contain only a few actions because there are no spare satellites available with which to perform replacement actions.

Table 4.12 Three satellite problem action set for states with no spares.

Action	Definition
$a_{s,1}$	Let the system run without intervention
$a_{s,2}$	Order one spare satellite
$a_{s,3}$	Order two spare satellites
$a_{s,4}$	Order three spare satellites

Table 4.13 lists the actions applicable to states with one spare satellite available at the decision epoch. The action sets summarized in this table are A_2, A_6, A_{10}, A_{14}, A_{18}, A_{22}, A_{26}, and A_{30}. Note that the replacement of each different satellite requires a separate action.

Table 4.13 Three satellite problem action set for states with one spare.

Action	Definition
$a_{s,1}$	Let the system run without intervention
$a_{s,2}$	Order one spare satellite
$a_{s,3}$	Order two spare satellites
$a_{s,4}$	Replace satellite 1
$a_{s,5}$	Replace satellite 1 and order one spare satellite
$a_{s,6}$	Replace satellite 1 and order two spare satellites
$a_{s,7}$	Replace satellite 1 and order three spare satellites
$a_{s,8}$	Replace satellite 2
$a_{s,9}$	Replace satellite 2 and order one spare satellite
$a_{s,10}$	Replace satellite 2 and order two spare satellites
$a_{s,11}$	Replace satellite 2 and order three spare satellites
$a_{s,12}$	Replace satellite 3
$a_{s,13}$	Replace satellite 3 and order one spare satellite
$a_{s,14}$	Replace satellite 3 and order two spare satellites
$a_{s,15}$	Replace satellite 3 and order three spare satellites

Table 4.14 lists the actions applicable to states with two spare satellites available at the decision epoch. The action sets summarized in this table are A_3, A_7, A_{11}, A_{15}, A_{19}, A_{23}, A_{27}, and A_{31}. This set of actions can call for the replacement of no satellites, a single satellite, or two satellites. Each possible combination of satellite replacement must be represented by a separate action.

Table 4.15 lists the actions applicable to states with three spare satellites available at the decision epoch. The action sets summarized in this table are A_4, A_8, A_{12}, A_{16}, A_{20}, A_{24}, A_{28}, and A_{32}. This set of actions allows for the replacement of any number of the satellites and must have separate actions representing each replacement strategy.

The transition probabilities are determined using the definitions provided in Section 3.4. For this example, the reliability parameters are the same as for the single-unit example and are extended to apply to all satellite in the constellation. These reliability parameters and the reliability probability values are summarized in Table 4.16 where λ_m, R_{satm}, and P_{slm} refer to the values specific to satellite m. For

Table 4.14 Three satellite problem action set for states with two spares.

Action	Definition
$a_{s,1}$	Let the system run without intervention
$a_{s,2}$	Order one spare satellite
$a_{s,3}$	Replace satellite 1
$a_{s,4}$	Replace satellite 1 and order one spare satellite
$a_{s,5}$	Replace satellite 1 and order two spare satellites
$a_{s,6}$	Replace satellite 2
$a_{s,7}$	Replace satellite 2 and order one spare satellite
$a_{s,8}$	Replace satellite 2 and order two spare satellites
$a_{s,9}$	Replace satellite 3
$a_{s,10}$	Replace satellite 3 and order one spare satellite
$a_{s,11}$	Replace satellite 3 and order two spare satellites
$a_{s,12}$	Replace satellites 1 and 2
$a_{s,13}$	Replace satellites 1 and 2 and order one spare satellite
$a_{s,14}$	Replace satellites 1 and 2 and order two spare satellites
$a_{s,15}$	Replace satellites 1 and 2 and order three spare satellites
$a_{s,16}$	Replace satellites 1 and 3
$a_{s,17}$	Replace satellites 1 and 3 and order one spare satellite
$a_{s,18}$	Replace satellites 1 and 3 and order two spare satellites
$a_{s,19}$	Replace satellites 1 and 3 and order three spare satellites
$a_{s,20}$	Replace satellites 2 and 3
$a_{s,21}$	Replace satellites 2 and 3 and order one spare satellite
$a_{s,22}$	Replace satellites 2 and 3 and order two spare satellites
$a_{s,23}$	Replace satellites 2 and 3 and order three spare satellites

this example each satellite uses the same parameter and reliability values although it is possible for different satellites to have different values.

The transition probabilities for the multi-unit example are computed as described in Section 3.4. Due to complexity of the system and the number of state transitions, extreme care must be taken to identify each event that can cause a transition from a state c to a state d $(c, d \in S)$. The probability of all such events is summed to determine the transition probability from state c to state d. For this example, the transition probabilities are too lengthy to be enumerated here, but an example of these transition probabilities are given in Table 4.17. This table shows the transition probabilities for choosing action $a_{3,15}$.

Table 4.15 Three satellite problem action set for states with three spares.

Action	Definition
$a_{s,1}$	Let the system run without intervention
$a_{s,2}$	Replace satellite 1
$a_{s,3}$	Replace satellite 1 and order one spare satellite
$a_{s,4}$	Replace satellite 2
$a_{s,5}$	Replace satellite 2 and order one spare satellite
$a_{s,6}$	Replace satellite 3
$a_{s,7}$	Replace satellite 3 and order one spare satellite
$a_{s,8}$	Replace satellites 1 and 2
$a_{s,9}$	Replace satellites 1 and 2 and order one spare satellite
$a_{s,10}$	Replace satellites 1 and 2 and order two spare satellites
$a_{s,11}$	Replace satellites 1 and 3
$a_{s,12}$	Replace satellites 1 and 3 and order one spare satellite
$a_{s,13}$	Replace satellites 1 and 3 and order two spare satellites
$a_{s,14}$	Replace satellites 2 and 3
$a_{s,15}$	Replace satellites 2 and 3 and order one spare satellite
$a_{s,16}$	Replace satellites 2 and 3 and order two spare satellites
$a_{s,17}$	Replace satellites $1, 2$ and 3
$a_{s,18}$	Replace satellites $1, 2$ and 3 and order one spare satellite
$a_{s,19}$	Replace satellites $1, 2$ and 3 and order two spare satellites
$a_{s,20}$	Replace satellites $1, 2$ and 3 and order three spare satellites

The multi-unit problem uses the same reward parameters used by the single-unit example, which are given in Table 4.6. The reward values are also too lengthy to be enumerated here, but an example is given in Table 4.18. This table provide the rewards for all of the possible actions for state 3.

This problem was solved using the Backward Induction Algorithm programmed in MATLAB®. Table 4.19 lists execution time characteristics for 30 runs of the program. The runs were conducted on a Dell® Inspiron 8100 with a 1 gigahertz Intel® Pentium® *III* processor, 256 megabytes of RAM, and using the Microsoft® Windows 2000 Professional operating system.

The policy derived from the algorithm is shown in Table 4.20. The table specifies which action should be selected depending on the system state during time

Table 4.16 Three satellite problem reliability values.

Reliability	Notation	Numerical Value
Mean lifetime of the satellite	λ_1^{-1}	40
Mean lifetime of the satellite	λ_2^{-1}	40
Mean lifetime of the satellite	λ_3^{-1}	40
Interval reliability of the satellite	R_{sat1}	0.9753
Interval reliability of the satellite	R_{sat2}	0.9753
Interval reliability of the satellite	R_{sat3}	0.9753
Probability of a successful launch	P_{sl1}	0.9500
Probability of a successful launch	P_{sl2}	0.9500
Probability of a successful launch	P_{sl3}	0.9500

epochs $1, \ldots, 40$ in order to minimize the total expected cost. For example, if the system is in state s_{31}, during any of the first 26 time epochs, action $A_{31,14}$ (replace the satellites 1 and 2 and order two new replacements) should be chosen. If the system is in state s_{31} during the time epochs 27 through 35, then action $A_{31,13}$ (replace the satellites 1 and 2 and order a new replacement) should be chosen. If the system is in state s_{31} during the time epochs 36 through 37, then action $A_{31,12}$ (replace the satellites 1 and 2 and do not order a replacement) should be chosen. If in state s_{31} during time epochs 38 or 39, action $A_{31,1}$ (let the system run without intervention) should be chosen. No decisions are made in time epoch 40 as there are no time periods evaluated after this epoch. Decisions are implemented at the beginning of the next time period and any decision made at time epoch 40 for this example would not be evaluated.

Table 4.21 shows the minimum expected total cost for the following the policy of Table 4.20. Note that the minimum expected cost is dependent upon the initial state of the system.

4.4 Analysis of the Multi-unit Example

The analysis of the multi-unit example closely follows the analysis the of the single-unit example. For satellite cost, launch cost, and holding cost the graphs of

Table 4.17 Transition probabilities for action $a_{3,15}$.

Probability	Definition	Value
$p_t(s_4\|s_3, a_{3,15})$	$(P_{sl1} \times P_{sl2} \times R_{sat3}) + ((1 - P_{sl1}) \times R_{sat1} \times P_{sl2} \times R_{sat3})$ $+(P_{sl1} \times (1 - P_{sl2}) \times R_{sat2} \times R_{sat3})$ $+((1 - P_{sl1}) \times (1 - P_{sl2}) \times R_{sat1} \times R_{sat2} \times R_{sat3})$	9.7290×10^{-1}
$p_t(s_8\|s_3, a_{3,15})$	$(P_{sl1} \times P_{sl2} \times (1 - R_{sat3}))$ $+((1 - P_{sl1}) \times R_{sat1} \times P_{sl2} \times (1 - R_{sat3}))$ $+(P_{sl1} \times (1 - P_{sl2}) \times R_{sat2} \times (1 - R_{sat3}))$ $+((1 - P_{sl1}) \times (1 - P_{sl2}) \times R_{sat1} \times R_{sat2} \times (1 - R_{sat3}))$	2.4629×10^{-2}
$p_t(s_{12}\|s_3, a_{3,15})$	$(P_{sl1} \times (1 - P_{sl2}) \times (1 - R_{sat2}) \times R_{sat3})$ $+((1 - P_{sl1}) \times (1 - P_{sl2}) \times R_{sat1} \times (1 - R_{sat2}) \times R_{sat3})$	1.2025×10^{-3}
$p_t(s_{16}\|s_3, a_{3,15})$	$((1 - P_{sl1}) \times (1 - R_{sat1}) \times P_{sl2} \times R_{sat3})$ $+((1 - P_{sl1}) \times (1 - P_{sl2}) \times (1 - R_{sat1}) \times R_{sat2} \times R_{sat3})$	1.2025×10^{-3}
$p_t(s_{20}\|s_3, a_{3,15})$	$(P_{sl1} \times (1 - P_{sl2}) \times (1 - R_{sat2}) \times (1 - R_{sat3}))$ $+((1 - P_{sl1}) \times (1 - P_{sl2}) \times R_{sat1}$ $\times(1 - R_{sat2}) \times (1 - R_{sat3}))$	3.0442×10^{-5}
$p_t(s_{24}\|s_3, a_{3,15})$	$((1 - P_{sl1}) \times (1 - R_{sat1}) \times P_{sl2} \times (1 - R_{sat3}))$ $+((1 - P_{sl1}) \times (1 - P_{sl2}) \times (1 - R_{sat1})$ $\times R_{sat2} \times (1 - R_{sat3}))$	3.0442×10^{-5}
$p_t(s_{28}\|s_3, a_{3,15})$	$(1 - P_{sl1}) \times (1 - P_{sl2}) \times (1 - R_{sat1})$ $\times(1 - R_{sat}2) \times R_{sat3}$	1.4864×10^{-6}
$p_t(s_{32}\|s_3, a_{3,15})$	$(1 - P_{sl1}) \times (1 - P_{sl2}) \times (1 - R_{sat1})$ $\times(1 - R_{sat2}) \times (1 - R_{sat3})$	3.7628×10^{-8}

the minimum expected total cost increase linearly and follow the same patterns as the graphs presented in Section 4.2. For these reasons, analysis of these parameters is not presented here.

The mean satellite lifetime for the multi-unit example is also varied from 1 quarter (3 months) to 80 quarters (20 years). For the multi-unit example, the minimum expected total cost is the equal for certain initial states. For example, all states with two operational satellites and no available spares (e.g. states s_5, s_9, s_{13}) all have the same minimum total expected cost. The optimal policy requires different actions for each state, but the cost are equivalent. Figure 4.6 shows the minimum expected total cost for 12 initial states. Figure 4.7 shows the minimum expected total cost for 12 other initial states. Each graph follows the same basic pattern so it is sufficient to

Table 4.18 Three satellite reward values example.

Reward	Definition	Value
$r_t(s_3, a_{3,1})$	$2 \times c_{hold}$	$\$100,000$
$r_t(s_3, a_{3,2})$	$2 \times c_{hold} + c_{sat}$	$\$50,100,000$
$r_t(s_3, a_{3,3})$	$c_{hold} + c_{launch}$	$\$55,050,000$
$r_t(s_3, a_{3,4})$	$c_{hold} + c_{launch} + c_{sat}$	$\$105,050,000$
$r_t(s_3, a_{3,5})$	$c_{hold} + c_{launch} + 2 \times c_{sat}$	$\$155,050,000$
$r_t(s_3, a_{3,6})$	$c_{hold} + c_{launch}$	$\$55,050,000$
$r_t(s_3, a_{3,7})$	$c_{hold} + c_{launch} + c_{sat}$	$\$105,050,000$
$r_t(s_3, a_{3,8})$	$c_{hold} + c_{launch} + 2 \times c_{sat}$	$\$155,050,000$
$r_t(s_3, a_{3,9})$	$c_{hold} + c_{launch}$	$\$55,050,000$
$r_t(s_3, a_{3,10})$	$c_{hold} + c_{launch} + c_{sat}$	$\$105,050,000$
$r_t(s_3, a_{3,11})$	$c_{hold} + c_{launch} + 2 \times c_{sat}$	$\$155,050,000$
$r_t(s_3, a_{3,12})$	$2 \times c_{launch}$	$\$110,000,000$
$r_t(s_3, a_{3,13})$	$2 \times c_{launch} + c_{sat}$	$\$160,000,000$
$r_t(s_3, a_{3,14})$	$2 \times c_{launch} + 2 \times c_{sat}$	$\$210,000,000$
$r_t(s_3, a_{3,15})$	$2 \times c_{launch} + 3 \times c_{sat}$	$\$260,000,000$
$r_t(s_3, a_{3,16})$	$2 \times c_{launch}$	$\$110,000,000$
$r_t(s_3, a_{3,17})$	$2 \times c_{launch} + c_{sat}$	$\$160,000,000$
$r_t(s_3, a_{3,18})$	$2 \times c_{launch} + 2 \times c_{sat}$	$\$210,000,000$
$r_t(s_3, a_{3,19})$	$2 \times c_{launch} + 3 \times c_{sat}$	$\$260,000,000$
$r_t(s_3, a_{3,20})$	$2 \times c_{launch}$	$\$110,000,000$
$r_t(s_3, a_{3,21})$	$2 \times c_{launch} + c_{sat}$	$\$160,000,000$
$r_t(s_3, a_{3,22})$	$2 \times c_{launch} + 2 \times c_{sat}$	$\$210,000,000$
$r_t(s_3, a_{3,23})$	$2 \times c_{launch} + 3 \times c_{sat}$	$\$260,000,000$

present only one graph in order to illustrate various arguments regarding the plot. Figures 4.6 and 4.7 show that, just as in the single-unit case, as the mean satellite lifetime increases, the minimum expected total cost decreases. It holds for this multi-unit example that the decrease in minimum expected total cost "flattens" out as the mean satellite lifetime approaches 80 quarters or 20 years. When the satellite mean lifetime reaches 42 quarters or 10.5 years, for each of the 32 possible starting states, the rate of change for the minimum expected total cost is less than 10 million dollars for every subsequent increase of one quarter to the mean satellite lifetime. For this example, a 10 million dollar change is less than a 2.5 percent change to the minimum expected total cost.

Table 4.19 Multi-unit run time characteristics.

Execution Statistic	Time (seconds)
Mean	13.7528
Mode	13.7600
Minimum	13.7190
Maximum	13.8500
Standard Deviation	0.0282

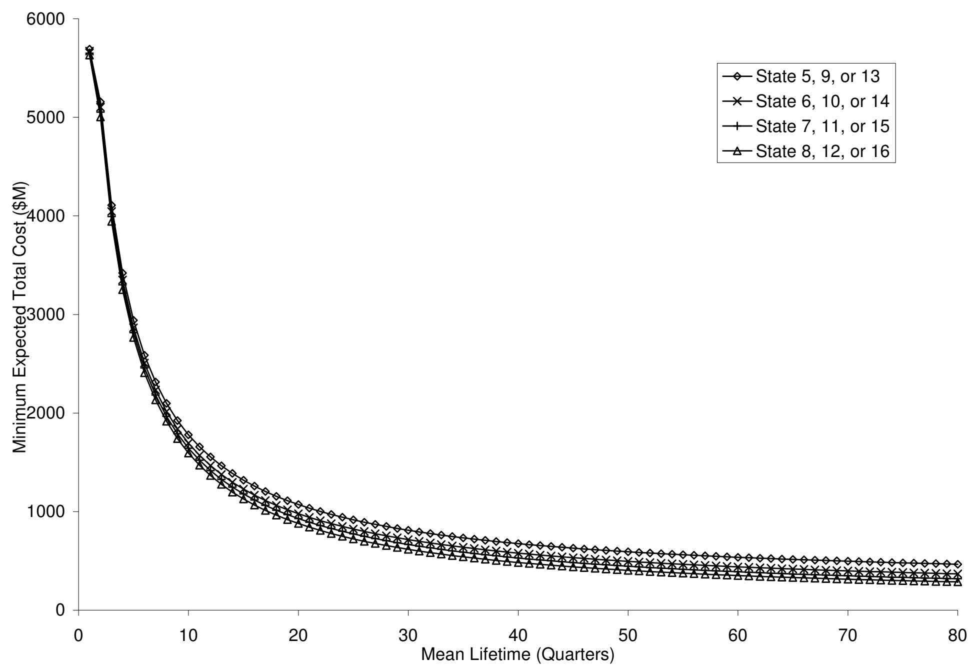

Figure 4.6 Varying the mean satellite lifetime over a 10-year time horizon.

A graph of the minimum expected total cost for a 10-year time horizon as the penalty cost is varied is shown in Figure 4.8. The penalty cost is varied from zero to 100 million dollars per quarter. This plot only shows the values for 12 of the initial states. Plots of the other initial states follow the pattern of this graph. Just as in the single-unit example, the figure shows a very low minimum expected total cost when the penalty cost is close to zero. When evaluating this case, the algorithm determines it is less expensive to pay penalties than to purchase and launch satellites to maintain the constellation. However, once the penalty cost reaches a relatively small value compared to the cost of buying and launching a satellite, the algorithm

Table 4.20 Multiple satellite policies.

State	Policy
s_1	2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0
s_2	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0
s_3	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0
s_4	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0
s_5	3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 0
s_6	13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 1, 1, 0
s_7	9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 1, 1, 0
s_8	6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 1, 1, 0
s_9	3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 0
s_{10}	9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 1, 1, 0
s_{11}	6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 1, 1, 0
s_{12}	4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 1, 1, 0
s_{13}	3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 0
s_{14}	5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 1, 1, 0
s_{15}	3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 1, 1, 0
s_{16}	2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 0
s_{17}	4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 3, 3, 3, 3, 3, 3, 3, 3, 3, 1, 1, 1, 1, 0
s_{18}	10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 9, 9, 9, 9, 9, 9, 9, 9, 9, 8, 8, 1, 1, 0
s_{19}	21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 1, 1, 0
s_{20}	14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 1, 1, 0
s_{21}	4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 3, 3, 3, 3, 3, 3, 3, 3, 3, 1, 1, 1, 1, 0
s_{22}	6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 5, 5, 5, 5, 5, 5, 5, 5, 5, 4, 4, 1, 1, 0
s_{23}	17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 16, 1, 1, 0
s_{24}	11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 1, 1, 0
s_{25}	4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 3, 3, 3, 3, 3, 3, 3, 3, 3, 1, 1, 1, 1, 0
s_{26}	6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 5, 5, 5, 5, 5, 5, 5, 5, 5, 4, 4, 1, 1, 0
s_{27}	13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 13, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 1, 1, 0
s_{28}	8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 1, 1, 0
s_{29}	4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 1, 1, 1, 1, 0
s_{30}	7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 6, 6, 6, 6, 6, 6, 6, 6, 6, 4, 4, 1, 1, 0
s_{31}	14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14, 13, 13, 13, 13, 13, 13, 13, 13, 13, 12, 12, 1, 1, 0
s_{32}	18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 18, 17, 17, 17, 17, 17, 17, 17, 17, 17, 17, 1, 1, 0

Table 4.21 Multiple satellite model minimum expected cost.

Initial State	Value ($M)	Initial State	Value ($M)
s_1	470.025	s_{17}	880.282
s_2	420.075	s_{18}	784.181
s_3	374.805	s_{19}	688.217
s_4	341.602	s_{20}	638.267
s_5	675.126	s_{21}	880.282
s_6	579.107	s_{22}	784.181
s_7	529.157	s_{23}	688.217
s_8	483.708	s_{24}	638.267
s_9	675.126	s_{25}	880.282
s_{10}	579.107	s_{26}	784.181
s_{11}	529.157	s_{27}	688.217
s_{12}	483.708	s_{28}	638.267
s_{13}	675.126	s_{29}	1085.443
s_{14}	579.107	s_{30}	989.309
s_{15}	529.157	s_{31}	893.265
s_{16}	483.708	s_{32}	797.356

determines it is more advantageous to maintain the satellite than to pay the penalty costs.

This chapter began with an example of a single-unit problem in order to demonstrate the use of Markov decision processes. Tables 4.9 and 4.10 showed the minimum expected total cost's dependence on the initial state of the system for the time horizon evaluated. Sensitivity analysis of problem parameters demonstrated the impact these values made on the minimum expected total cost. Next, a multi-unit, or constellation, example was given. Similar results and sensitivity analysis followed the example. Chapter 5 presents a summary of this thesis, discusses insights gained during the research, explains the contributions, and details future work in the area of this thesis.

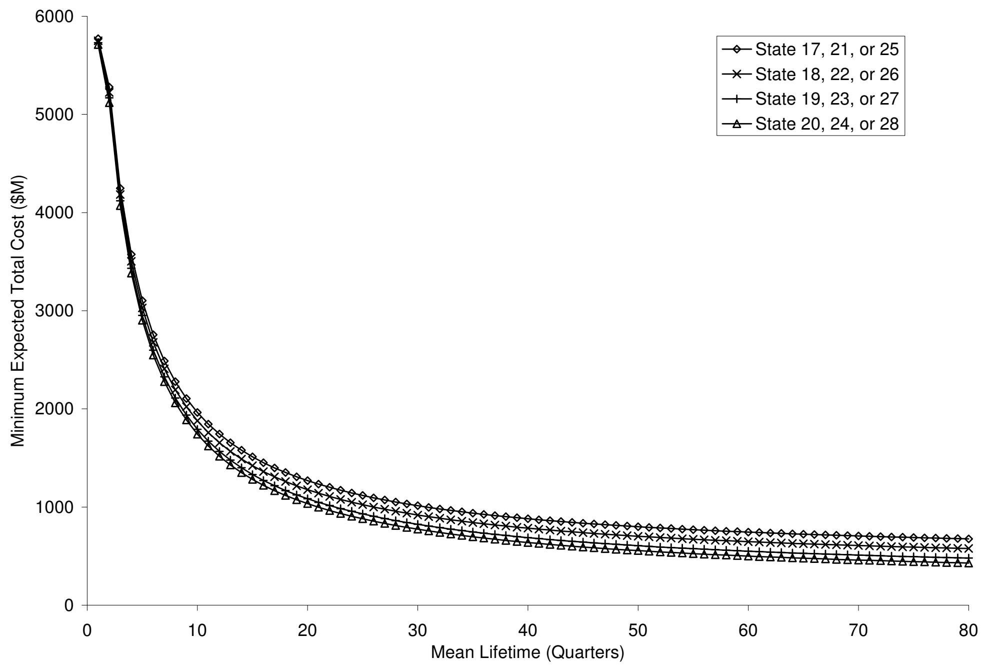

Figure 4.7 Varying the mean satellite lifetime over a 10-year time horizon.

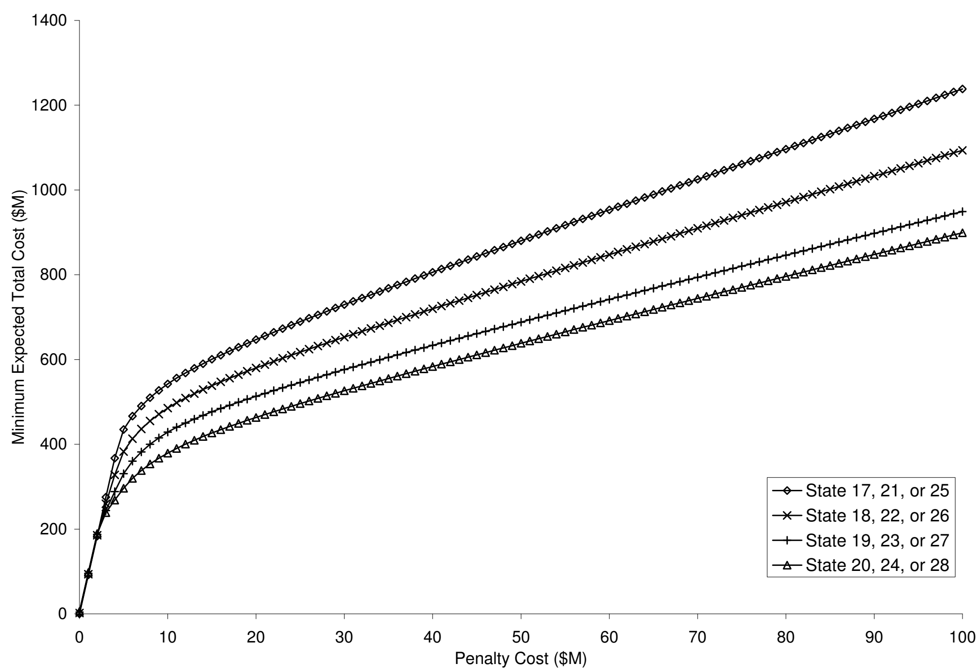

Figure 4.8 Varying the penalty cost over a 10-year time horizon.

5. Conclusions and Future Research

The primary objectives of this thesis were to create an analytical model of satellite constellations, to find optimal replacement policies that minimize the expected total cost of maintaining the constellations, and to study how changes to model parameters affect the policy and the minimum expected total cost. The importance of analytically modeling satellite constellations via Markov decision processes is that a provably optimal replacement policy can be derived. Knowing the optimal replacement policy aides a decision maker in making informed decisions regarding the maintenance of a constellation. While following the policy does not guarantee that the minimum cost is achieved, this course of action ideally has a higher likelihood of achieving the minimum cost than any other strategy. Sensitivity analysis on the model parameters provided a way to evaluate how the optimal value is affected by changes to the problem parameters. If the optimal value changes dramatically with small changes to a parameter, it will be important to ensure that accurate information about the parameter is included in the model.

In order to determine an optimal replacement policy, an analytical model of the satellite constellation was created and solved using Markov decision processes (stochastic dynamic programming) to find the optimal replacement policy. The third chapter provided a review of Markov decision processes, stated the assumptions made in modeling the satellite constellations, and formulated models for single-unit and multi-unit systems. A discussion of linear programming formulations extended the review of Markov decision processes. Several assumptions about satellites and their operation were made in order to model satellite constellations as Markov decision processes and to avoid the problem of state space explosion. These assumptions are generally mild, in that they do not greatly modify the characteristics of many of the existing satellite constellations. The single-unit model was formulated for the purpose of demonstrating Markov decision process modeling for a satellite constel-

lation. For the single-unit model it was possible to explicitly identify the states, actions, transition probabilities, and rewards. This aided the understanding of the multi-unit model. The multi-unit model was defined by presenting rules that were used to determine the system states, actions, transitions probabilities, and rewards. Equations for computing the number of states and the number of actions were provided to determine the size of the state space as a function of the number of satellites in the constellation.

Chapter 4 specified model parameters for both single-unit and multi-unit examples. The examples of each problem were followed by sensitivity analysis of model parameters. Model parameters were specified by determining realistic values from actual space systems. The multi-unit example used the same parameter values as the single-unit example. The parameter values were assumed to be identical for satellite in the constellation. A constellation of three active satellites was assumed for the multi-unit example. Analysis of both the single-unit and multi-unit examples included sensitivity analysis of the model parameters and interpretation of these analyses.

The main contribution of this research was the development of an analytical model used to determine the optimal replacement policy for satellite constellations. This analytical model serves as a framework upon which a more detailed model can be based. The use of an analytical model provides a provably optimal replacement policy for the given assumptions and offers an alternative to potentially time-consuming simulation. High resolution simulation modeling is often used to analyze various policies for satellite constellations. However, simulation cannot prove the optimality of a given policy. Policies found by the analytical methodology of this thesis can be used to determine budgets for maintaining the constellation. Because the models do not assume budgetary constraints, the policies found by using Markov decision processes provide the minimum expected total cost over the entire time-horizon being evaluated. Therefore, by considering the actions of the optimal policy for a given

time period t, a realistic budgetary value can be derived. The sensitivity analysis of the model parameters can be used to determine where small changes to model parameters could have a large effect on the minimum expected total cost.

For the purpose of analytical tractability, several assumptions were employed regarding the satellites and their operation. The model can be extended by relaxing some of these assumptions. The first important assumption to be relaxed is that of exponentially distributed satellite lifetimes. One alternative is to use phase-type (PH) distributions to model the satellite lifetime distribution. PH-distributions can be used to represent general distributions by the convolution of multiple exponential distributions [1]. Well-known examples of PH-distributions include the Erlang and Coxian distributions. The main feature of PH distributions that make them of value in this context is that they maintain the Markovian (memoryless) property [24] and are able to approximate any probability distribution. However, such approximations come at a computational expense due the additional number of required system states for the underlying Markov chain. Hence, accuracy must be balanced with this expense.

The models assumed that a satellite is either operational or non-operational. Allowing the level of the satellite's health to be represented by several states would more accurately represent real-world scenarios. The different states can hypothetically represent the amount of remaining useful life for a satellite. Hence, if some penalty cost is assessed for operating in a degraded state, the model could ideally determine the degradation point at which satellites should be replaced to minimize the expected total cost over the time-horizon evaluated. Increasing the number of states representing the condition of a satellite greatly increases not only the state space of the entire problem, but also the number of state transitions that must be determined. The computational power and available memory are important considerations in determining how many levels of satellite health should be represented.

Allowing multiple satellites to be launched on a single launcher is another possible improvement. However, the ability to do this depends on the satellite system being evaluated. For example, it is unrealistic to think that multiple heavy satellites, such as Milstar, could be launched on a single launcher. Relaxing this assumption would be valid only for certain satellite systems. However, the model is robust enough to be extended to such situations when appropriate.

Modeling the capability to have on-orbit-spare satellites is another possible improvement. There are actually two distinct types of on-orbit-spares that could be modeled. Hot spares would be activated and ready to perform the mission as soon a the primary satellite could not longer do so. A cold spare would be in a standby mode and would need to be activated in the event that the primary satellite could no longer perform the mission. A hot spare would incur faster degradation than a cold spare, but both would incur less degradation than an active satellite. Including the use of on-orbit-spares would allow this capability to be evaluated to determine if it was a cost effective measure to use. It is likely that on-orbit-spares would be used when the penalty cost of a mission failure is very high.

This thesis suggested that the penalty cost should be determined through the use of a utility or value function. Further work in area of determining the utility of a satellite constellation is necessary to properly use this aspect of the model (cf. [33]). Furthermore, the holding cost could be represented as a utility function. When a satellite is built and held in storage, advances in technology and design are not incorporated into the satellite prior to its launch unless costly retrofits are made. These changes can improve the capabilities of the satellite and extend it's lifetime. Therefore, it would be more realistic to account for this opportunity cost in future analyses.

Another generalization of the model is to allow the lead time for replacement orders to be a random variable. This would allow the production time of a satellite to span across multiple decision epochs. The production time for some satellites is

longer than three months. This change would make the models more realistic by requiring that replacement satellites be ordered before the expected failure of active satellites. This may result in a replacement not being available when needed, or in paying large holding costs for replacement satellite built well before the time they are required.

Another area of study is the availability of launchers and launch facilities. Under normal operating conditions, a sufficient number of launchers and launch facilities are available to launch satellites into orbit given proper planning and scheduling. These conditions are assumed to exist in the proposed models. In a surge period, or when rapid replenishment of a constellation is required, this assumption is not likely to hold true. The models presented here are only concerned with a single constellation. An in-depth study of launcher and launch facility availability would need to incorporate the priority given to a particular constellation to determine the availability of resources to that constellation.

A major challenge during the course of this thesis was determining the scope of this initial effort. The more detail incorporated into an analytical model increase the state space of the problem. Determining the level of detail at which to model satellite constellations required balancing the degree of reality in model outputs and computational complexity of finding results. In order to show the value of an analytical modeling approach sufficient problem assumptions were required to allow the model to be presented in a clear manner. Relaxation of these assumptions is fertile ground for future research of this problem.

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14. ABSTRACT

This work considers the problem of finding optimal replacement policies that minimize the expected total cost of maintaining a satellite constellation. The problem is modeled using discrete-time Markov decision processes to determine the replacement policy by allowing the satellite constellation to be in one of a finite number of states at each decision epoch. The constellation stochastically transitions at each time step from one state to another as determined by a set of transition probabilities. At each decision epoch, a decision maker chooses an action from a set of allowable actions for the current system state. A cost associated with each possible action is determined by the number of satellites purchased, launched, or held in storage, as well as the operational capability of the constellation. The system is evaluated for a given time horizon using the standard Policy Evaluation Algorithm of Markov decision processes (stochastic dynamic programming) to determine the optimal replacement policy and the minimum expected total cost. Example problems using notional data are presented to demonstrate the solution procedures. Sensitivity analysis of problem parameters is performed to investigate their impact on the minimum expected total cost of operating the constellation over a specified time horizon.

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